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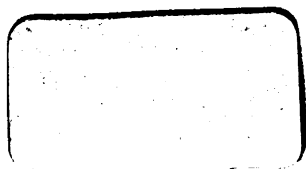
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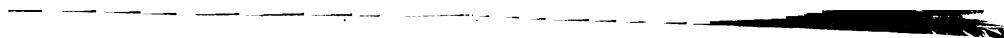
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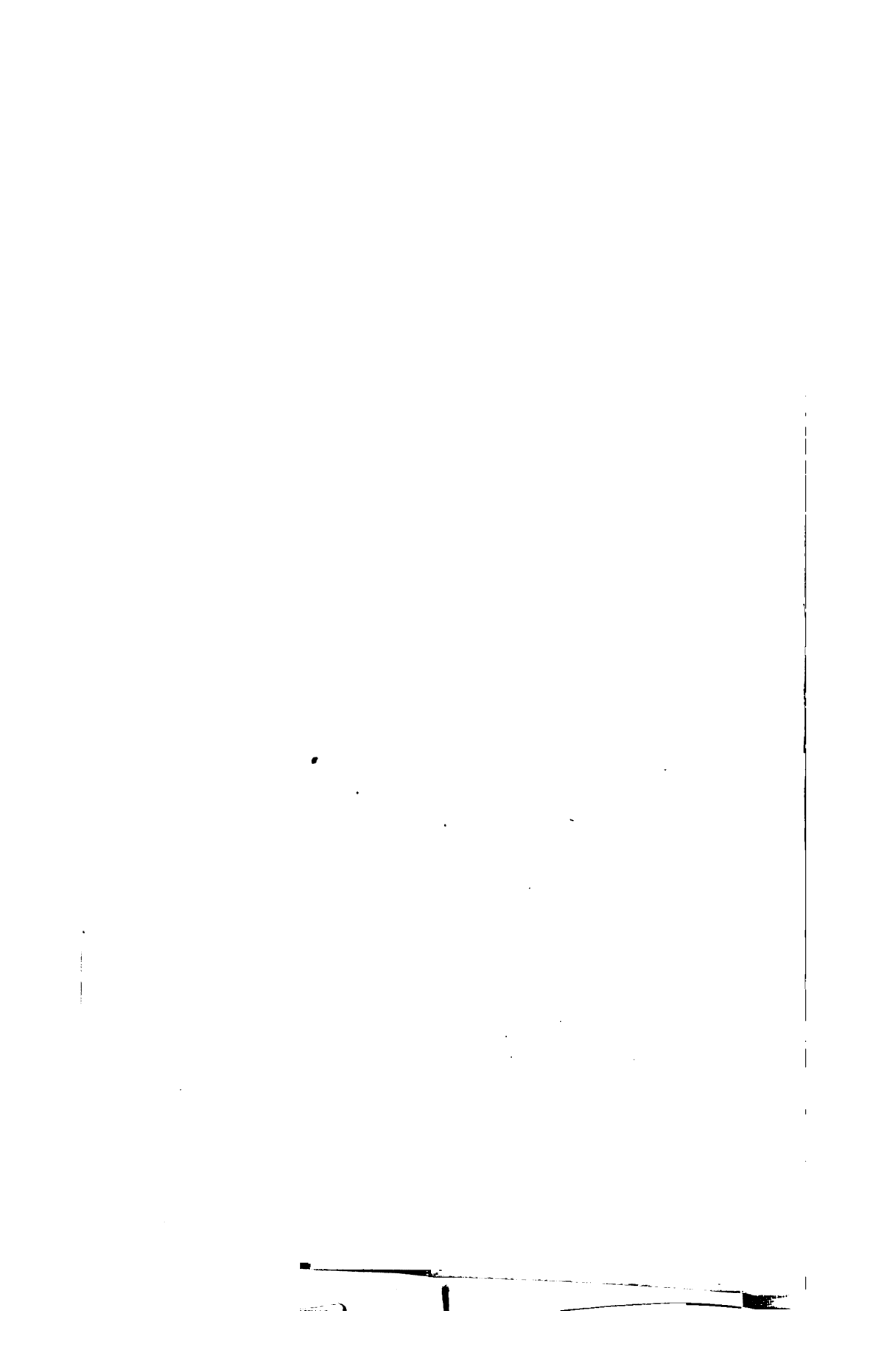
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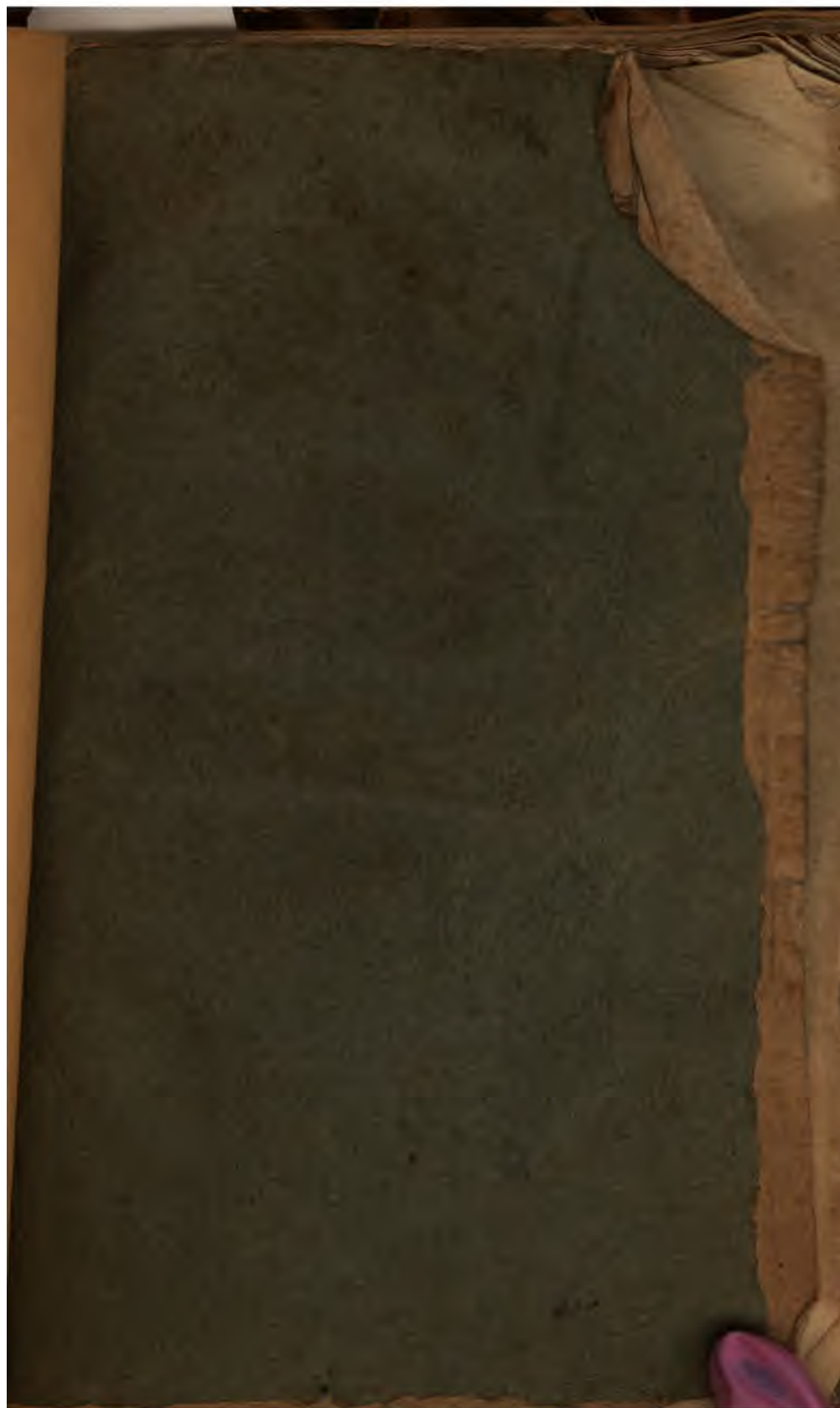




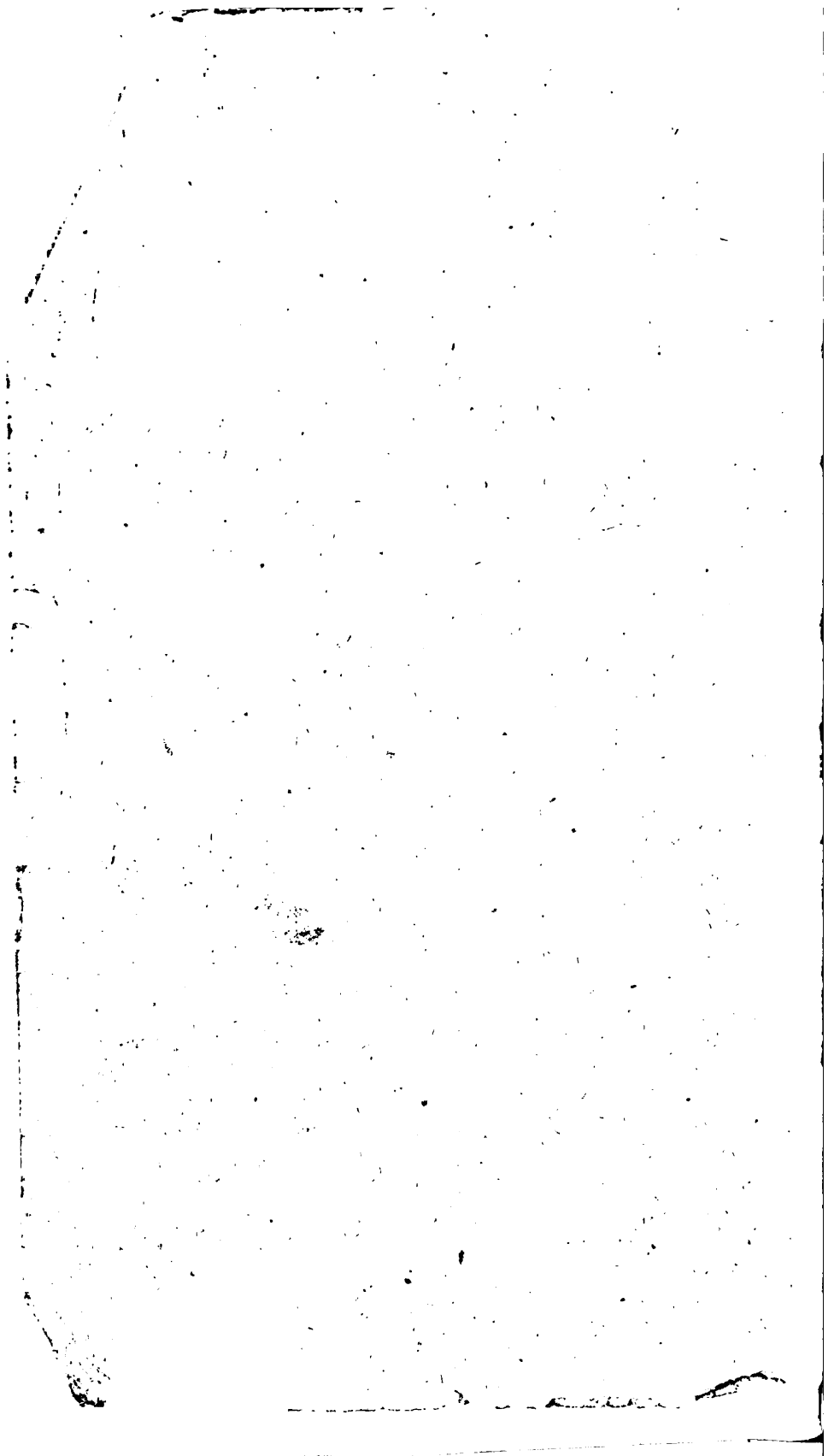












THE  
P R I N C I P L E S  
O F  
B R I D G E S:

CONTAINING THE  
MATHEMATICAL DEMONSTRATIONS

O F

The PROPERTIES of the ARCHES, the  
THICKNESS of the PIERS, the FORCE  
of the WATER against them, &c.

TOGETHER WITH

PRACTICAL OBSERVATIONS and DIRECTIONS  
drawn from the whole.

---

By C H A. H U T T O N,  
MATHEMATICIAN.

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NEWCASTLE:

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*Readwell fund.*

## P R E F A C E.

*A* Large and elegant bridge, forming a way over a broad and rapid river, is justly esteemed one of the noblest pieces of mechanism that man is capable of performing. And the usefulness of an art which, at the same time that it connects distant shores by a way over the deep and rapid waters, also allows those waters and their navigation to pass smooth and uninterrupted, renders all probable attempts to advance the theory or practice of it, highly deserving the encouragement of the public.

This little book is offered as an attempt towards the perfection of the theory of this art, in which the properties, dimensions, proportions, and other relations of the various parts of a bridge, are strictly demonstrated, and clearly illustrated by various examples. It is divided into five sections: the 1st treats on the projects of bridges, containing a regular detail of the various circumstances and considerations that are cognizable in such projects: The 2d treats on arches, demonstrating their various properties, with the relations between their intrados and extrados, and clearly distinguishes the most preferable curves to be used in a bridge; the first two or three propositions being instituted after the manner of two or three done by Mr. Emerson in his *Fluxions and Mechanics*: The 3d section treats on the piers, demonstrating their thickness necessary for supporting any kind of an arch, springing at any height, and that both when part of the pier is supposed to be immersed in water, and when otherwise: The 4th demonstrates the force of the water against the end or face of the pier, considered as of different forms; with the best form for dividing the stream, &c. and to it is added a table shewing the several heights of the fall of the water under the arches, arising from its velocity and the obstruction of the piers; as it was composed by Tho. Wright, Esq; of Auckland, in the county of Durham, who informs me it is part of a work in which he has spent much time, and with which he intends to favour the public: And the 5th and last section contains a dictionary of the most material terms peculiar to the subject;

in

*in which many practical observations and directions are given, which could not be so regularly nor properly introduced into the former sections: The whole, it is presumed, containing full directions for constituting and adapting to one another, the several essential parts of a bridge, so as to make it the strongest; and the most convenient, both for the passage over and under it; that the situation and other circumstances will possibly admit: not indeed for the actual methods of disposing the stones, making of mortar, or the external ornaments, &c. those things I do not descend to, but leave to the discretion of the practical architect, as being no part of the plan of my undertaking; and for the same reason also I have given no views of bridges, but only prints of such parts or figures as are necessary in explaining the elementary parts of the subject.*

*As my profession is not that of an architect, very probably I should never have turned my thoughts to this subject, so as to address the public upon it, had it not been from the occasion of an accident in that part of the country in which I reside, viz. the fall of Newcastle and other bridges on the river Tyne on the 17th of november 1771, occasioned by a high flood which rose about 9 feet higher at Newcastle than the usual spring tides do.—And this occasion having furnished me with many opportunities of hearing and seeing very absurd things advanced on the subject in general, I thought the demonstrations of the relations of the essential parts of a bridge, would not be unacceptable to those architects and others who may be capable of perceiving the force of them, and whose ignorance may not have prejudiced them against things which they do not understand.*

*In the 4th section there is one thing forgotten to be remarked, viz. That in determining the best form of the end of the pier to be a right-lined triangle, the water was supposed to strike every part of it with the same velocity: had the variably increased velocity been used, the form of the ends would come out a little curved; but as the increase of the velocity in the best bridges is very small, the difference in them is quite imperceptible.*

THE

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T H E

# PRINCIPLES

O F

## STONE BRIDGES.

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### SECTION I.

*Of the Projects of Bridges, with the Design,  
Estimate, &c.*

**W**HEN a bridge is deemed necessary to be built over a river, the first consideration is the place of it; or what particular situation will contain a maximum of the advantages over the disadvantages.

In agitating this most important question, every circumstance, certain and probable, attending or likely to attend the bridge, should be separately, minutely, and impartially stated and examined; and the advantage or disadvantage of it rated at a value proportioned to it: then the difference between the whole advantages and

B

dif-

disadvantages, will be the neat value of that particular situation for which the calculation is made. And by doing the same for any other situations, all their neat values will be found, and of consequence the most preferable situation among them.—Or, in a competition between two places, if each one's advantage over the other be estimated or valued in every circumstance attending them, the sums of their advantages will shew whether of them is the better. And the same being done for this and a third, and so on, the best situation of all will be obtained.

In this estimation, a great number of particulars must be included; and nothing omitted that can be found to make a part of the consideration.

Among these, the situation of the town or place for the convenience of which the bridge is chiefly to be made, will naturally produce a particular of the first consequence; and a great many others ought to be sacrificed to it. If possible, the bridge should be placed where there can conveniently be opened and made passages or streets from the ends of it in every direction, and especially one as nearly in the direction of the bridge itself as possible, tending towards the body of the town, without narrows or crooked windings, and easily communicating with the chief streets, thoroughfares, &c.—And here every

Sect. I. *The Projects of Bridges, &c.* 3

every person, in judging of this, should divest himself of all partial regards or attachments whatever; think and determine for the good of the whole only, and for posterity as well as the present.

The banks or declivities towards the river are also of particular concern, as they affect the conveniency of the passage to and from the bridge, or determine the height of it, upon which in a great measure depends the expence.

The breadth of the river, the navigation upon it, and the quantity of water to be passed, or the velocity and depth of the stream, form also considerations of great moment; as they determine the bridge to be higher or lower, longer or shorter. However, in most cases, a wide part of the river ought rather to be chosen than a narrow one, especially if it is subject to great tides or floods; for, the increased velocity of the stream in the narrow part, being again augmented by the farther contraction of the breadth, by the piers of the bridge, will both incommode the navigation through the arches, and undermine the piers and endanger the whole bridge.

The nature of the bed of the river is also of great concern, it having a great influence on the expence; as upon it, and the depth and velocity



#### 4      *The PRINCIPLES of BRIDGES.*

of the stream, depend the manner of laying the foundations, and building the piers.

These are the chief and capital articles of consideration, and which will branch themselves out into other dependent ones, and so lead to the required estimate of the whole,

HAVING resolved on the place, the next considerations are the form, the estimate of the expence, and the manner of execution.

With respect to the form ; strength, utility, and beauty ought to be regarded and united ; the chief part of which lies in the arches. The form of the arches will depend on their height and span ; and the height on that of the water, the navigation, and the adjacent banks. They ought to be made so high, as that they may easily transmit the water at its greatest height either from tides or floods ; and their height and figure ought also to be such as will easily allow of a convenient passage of the craft through them. This and the disposition of it above, so as to render the passage over it also convenient, make up its utility.—Having fixed the heights of the arches, their spans are still necessary for determining their figure. Their spans will be known by dividing the whole breadth of the river into a convenient number of arches and piers, allowing at least the necessary thickness of the  
the

the piers out of the whole. In fixing on the number of arches, take always an odd number, and rather take few and large ones than many and smaller, if convenient: For thus you will have not only fewer foundations and piers to make, with fewer arches and centers, which will produce great savings in the expence, but the arches themselves will also require much less materials and workmanship, and allow of more and better passage for the water and craft through them; and will appear at the same time more noble and beautiful, especially if constructed in elliptical, or in cycloidal forms: for the truth of which it may be sufficient to refer to that noble and elegant bridge lately built at Blackfriars, London, by Mr. Mylne. And here I can't help remarking that the Gentleman who, a few years since in a pamphlet on the Principles of Bridges, censured Mr. Mylne and Mr. Muller concerning elliptic arches, has very much exposed himself, and absurdly criticises them through his own want of mathematical knowledge, which he somewhere in the same pamphlet affects to despise. He brings to my mind an expression of (I think) Mr. Henry Fielding somewhere in his works, That a person does not speak the worse on a subject for knowing something about it. I do not however make this remark through any particular disrespect for this Gentleman, concerning whom I know nothing farther, any more than I do about the other two Gentlemen, but only to prevent

prevent others from being prejudiced and misled by the authority of his *ipse dixit*.—If the top of the bridge be a straight horizontal line, let the arches be made all of a size; if it be a little lower at the ends than the middle, the arches must proportionally decrease from the middle towards the ends; but if higher at the ends than the middle, let them increase towards the ends. A choice of the most convenient arches is to be made from the 4th and 5th propositions, where their several properties, &c. are demonstrated and pointed out: Among them, the elliptic, cycloidal, and equilibrial arch in prop. 5, will generally claim the preference, both on account of their strength, beauty, and cheapness or saving in materials and labour: Other particulars also concerning them may be seen under the word ARCH in the Dictionary in the last section. And as the choice of the arch is of so great moment, let no person, either through ignorance or indolence, prefer a worse arch because it may seem to him easier to construct; for he would very ill deserve the name or employment of an Architect, who is incapable of rendering the exact construction of these curves easy and familiar to himself; but if, by chance, a Bridge-builder should be employed who is incapable of doing that, he ought at least to be endowed with such a share of honesty as to procure some person to go through the calculations which he cannot make for himself.

Next

Next find what thickness at the keystone or top will be necessary for the arches. For which see the word KEYSTONE in the Dictionary in the last section.

Having thus obtained all the parts of the arches, with the height of the piers, the necessary thickness of the piers themselves are next to be computed by prop. 10.

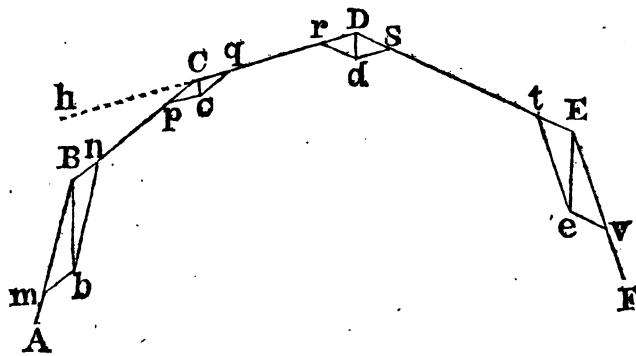
This done, the chief and material requisites are found; the elevation and plans of the design can then be drawn, and the calculations of the expence from thence made, including the foundations, with such ornamental or accidental appendages as shall be thought fit; which I shall leave to the discretion of the Practical Architect, as being no part of the plan of my undertaking, together with the practical methods of carrying the design into execution. I shall however, in the Dictionary in the last section, not only describe the terms, parts, machines, &c. but also speak of their dimensions, properties, and any thing else material belonging to them; and to which therefore I from hence refer for more explicit information in each particular article, as well as to these immediately following propositions, in which the theory of the arches, piers, &c. are fully and strictly demonstrated.

## SECTION II.

*Of the Arches.*

## PROPOSITION I.

**L**ET there be any number of lines AB, BC, CD, DE, &c. all in the same vertical plane, connected together and moveable about the joints or angles A, B, C, D, E, F; the two extreme points A and F being fixed: It is required to find the proportions of the weights to be laid upon the angles B, C, D, &c. so that the whole may remain in equilibrium.

*Solution.*

FROM the several angles having drawn the lines Bb, Cc, Dd, &c. perpendicular to the horizon; about them, as diagonals, constitute paral-

parallelograms such, that those sides of each two that are upon the same one of the given lines, may be equal to each other; viz. having made one parallelogram  $mn$ , take  $Cp = Bn$ , and form the parallelogram  $pq$ ; then take  $Dr = Cq$ , and make the parallelogram  $rs$ ; and take  $Et = Ds$ , and form the parallelogram  $tv$ ; and so on: Then the said vertical diagonals  $Bb$ ,  $Cc$ ,  $Dd$ ,  $Ee$ , &c. of those parallelograms, will be proportional to the weights, as required.

*Demonstration.*

By the resolution of forces, each of the weights or forces  $Bb$ ,  $Cc$ ,  $Dd$ , &c. in the diagonals of the parallelograms, is equal to, and may be resolved into two forces expressed by two adjacent sides of the parallelogram; viz. the force  $Bb$  will be resolved into the two forces  $Bm$ ,  $Bn$ , and in those directions; the force  $Cc$  into the two forces  $Cp$ ,  $Cq$ , and in those directions; the force  $Dd$  into the two forces  $Dr$ ,  $Ds$ , and in those directions; and so on: Then, since two forces that are equal, and in opposite directions, do mutually balance each other; therefore the several pairs of forces  $Bn$  and  $Cp$ ,  $Cq$  and  $Dr$ ,  $Ds$  and  $Et$ , &c. being equal and opposite, by the construction, do mutually destroy or balance each other; and the extreme forces  $Bm$ ,  $Ev$ , are balanced by the opposite resistances of the fixed points  $A$ ,  $F$ . Wherefore there is no force

C to

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to change the position of any one of the lines, and consequently they will all remain in equilibrium. *Q.E.D.*

*Corollary.*

HENCE, if one of the weights and the positions of all the lines be given, all the other weights may be found.

PROPOSITION II.

*IF any number of lines, that are connected together and moveable about the points of connection, be kept in equilibrium by weights laid upon the angles, as in the last proposition: Then will the weight on any angle C be universally as  $\frac{\text{fine of the } \angle BCD}{s. \angle BCc \times s. \angle cCD}$ ; that is, directly as the sine of that angle, and reciprocally as the sines of the two parts or angles into which that angle is divided by a line drawn through it perpendicular to the horizon.*

*Demonstration.*

By the last proposition the weights are as Bb, Cc, Dd, &c. when  $Bn = pC$ ,  $Cq = rD$ ,  $Ds = tE$ , &c. But, since the angle ABb is = the angle Bbn,

Sect. II.

*Of the Arches.*

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Bbn, and the angle B Cc = the angle Ccq, &c. as being always the alternate angles made by a line cutting two other parallel lines; also the sine of the  $\angle ABC = s. \angle Bnb$ , and  $s. \angle BCD = s. \angle Cqc$ , as being supplements one to another; by plane trigonometry we shall have

$$(Bn =) \frac{Bb \times s. \angle ABb}{s. \angle ABC} = (Cp =) \frac{Cc \times s. \angle cCD}{s. \angle BCD},$$

$$(Cq =) \frac{Cc \times s. \angle BCc}{s. \angle BCD} = (Dr =) \frac{Dd \times s. \angle dDE}{s. \angle CDE},$$

$$(Ds =) \frac{Dd \times s. \angle CDd}{s. \angle CDE} = (Et =) \frac{Ee \times s. \angle eEF}{s. \angle DEF},$$

&c.

Hence

$$Bb : Cc :: \frac{s. \angle ABC}{s. \angle ABb} : \frac{s. \angle BCD}{s. \angle cCD},$$

$$Cc : Dd :: \frac{s. \angle BCD}{s. \angle BCc} : \frac{s. \angle CDE}{s. \angle dDE},$$

$$Dd : Ee :: \frac{s. \angle CDE}{s. \angle CDd} : \frac{s. \angle DEF}{s. \angle eEF},$$

&c.

Or, by dividing the latter terms of the first of these proportions each by  $s. \angle bBC$ , and then compounding together two of the proportions, then three of them, &c. striking out the common factors, and observing that the  $s. \angle bBC$  is =  $s. \angle BCc$ , the  $s. \angle cCD = s. \angle CDd$ , &c. we shall have

C 2

Bb



$$Bb : Cc :: \frac{s. \angle ABC}{s. \angle ABb \times s. \angle bBC} : \frac{s. \angle BCD}{s. \angle BCc \times s. \angle cCD},$$

$$Bb : Dd :: \frac{s. \angle ABC}{s. \angle ABb \times s. \angle bBC} : \frac{s. \angle CDE}{s. \angle CDd \times s. \angle dDE},$$

$$Bb : Ee :: \frac{s. \angle ABC}{s. \angle ABb \times s. \angle bBC} : \frac{s. \angle DEF}{s. \angle DEe \times s. \angle eEF},$$

&c.

*Q.E.D.**Otherwise.*

SINCE  $Cp$  or  $Bn : Bm$  or  $nb :: s. \angle Bbn$   
or  $s. \angle ABb : s. \angle bBC$  or  $s. \angle BCc ::$

$$\frac{I}{s. \angle BCc} : \frac{I}{s. \angle ABb},$$

and  $Cp$  or  $qc : Cq$  or  $Dr :: s. \angle cCq$  or  
 $s. \angle CDd : s. \angle Cc q$  or  $s. \angle BCc ::$

$$\frac{I}{s. \angle BCc} : \frac{I}{s. \angle CDd},$$

it is clear that  $Cp$  is as  $\frac{I}{s. \angle BCc}$ ; that is,  
the forces  $mB$ ,  $pC$ ,  $rD$ , &c. are reciprocally as  
the fines of the angles which they make with  
the vertical line.

And since  $Cc$  is  $= \frac{Cp \times s. \angle Cpc}{s. \angle Ccp} =$   
 $\frac{Cp \times s. \angle BCD}{s. \angle cCD}$ ; therefore any force  $Cc$  is  
as  $\frac{s. \angle BCD}{s. \angle cCB \times s. \angle cCD}$ . *Q.E.D.*

*Corol-*

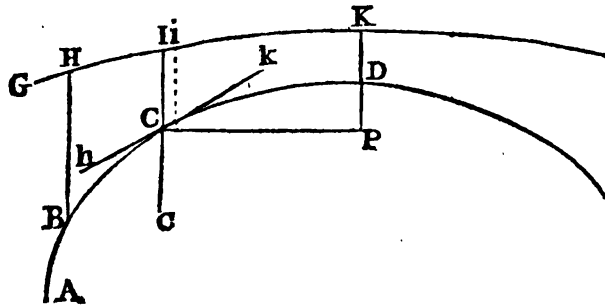
*Corollary.*

If DC be produced to h; the sine of the  $\angle hCB$  being = to the sine of its supplement BCD, the weight or force Cc will be as  $\frac{s. \angle hCB}{s. \angle BCc \times \angle cCD}$ ; which three angles together make up two right angles.

PROPO-

## PROPOSITION III.

TO find the proportion of the height of the wall above every point of an arch of equilibration: That is, if  $GHIK$  be the top of a wall supported by an arch  $ABCD$ ; it is required to find the proportion of the perpendiculars  $BE$ ,  $CI$ , &c. so that all the parts of the arch may be kept in equilibrium from falling, by the weight or pressure of the superincumbent wall.

*Solution.*

THE lines of equilibration in the former propositions being imagined to become indefinitely small, they will constitute a curve of equilibration, and the weights will press upon every point of it, and will be respectively equal to the perpendiculars

pendiculars BH, CI, &c. drawn into their respective breadths, supposing them to be indefinitely narrow parallelograms: Also the angle hCB will become the angle of contact formed by the tangent and curve, whose sine is equal to the angle itself or its measure, and the angles cCB and cCD become equal to the angles cCh, cCk, or equal to the angles ICk, ICh, whose sines are equal, because the angles are supplements to each other. These values being substituted in the expression in the corollary to the last proposition, we shall have the force Cc or parallelogram Ci as  $\frac{\text{the angle hCB}}{s. \angle hCI}$ .

or as  $\frac{\text{the } \angle kCD}{s. \angle kCI}$ .

Now supposing these narrow parallelograms to stand upon indefinitely small equal parts of the arch, their breadths will be directly as the  $s. \angle kCI$  and inversely as radius; hence the parallelogram IC  $\propto s. \angle kCI$  is as  $\frac{\text{the } \angle kCD}{s. \angle kCI}$ ,

and consequently the altitude IC as  $\frac{\text{the } \angle kCD}{s. \angle kCI}$ ,

or as the  $\angle kCD \times \text{secant } \angle kCI$ ; CP being perpendicular to CI, and the radius all along equal to unity.

But the angle of contact kCD is as the curvature of the arch, and that again is inversely  
as

as the radius of curvature ; wherefore  $IC$  is

$$\text{as } \frac{1}{R \times \text{s. } \angle KCI}, \text{ or as } \frac{\text{sec. } \angle KCI}{R}, \text{ putting}$$

$R$  for the radius of curvature to the point  $C$  ; that is, the height of the wall above any point, is reciprocally as the radius of curvature and cube of the sine of the angle in which the vertical line cuts the curve in that point, or reciprocally as the radius of curvature and directly as the cube of the secant of the curve's inclination to the horizon.

*Corollary 1.*

HENCE, if the form of the arch, or nature of the curve  $ABCD$  be given, the form of the line  $GHIK$  bounding the top of the wall or forming the extrados, may be found so, that  $ABCD$  shall be an arch of equilibration, or be in equilibrium in all its parts by the pressure of the wall.

For, since the arch is given, the radius of curvature and position of the tangent at every point of it will be given, and consequently the proportions of the verticals  $BH$ ,  $CI$ , &c. And by assuming one of them, or making it equal to an assigned length, the rest will be found from it ; and then the line  $GHI$  &c. may be drawn through the extremities of them all.

*Corol-*

*Corollary 2.*

AND if the line GHIK, forming the top of the wall be given, the curve of equilibration ABCD may be found. And the manner of finding them both, the one from the other, we shall teach in the two following propositions.

*Corollary 3.*

IF the arch ABCD be a circle; the radius of curvature will be constant, and the angle kCP always measured by the arc DC, supposing D the vertex of the curve; and then CI will be every-where as the cube of the secant of the arc DC.

## PROPOSITION IV.

*HAVING given the Intrados, to find the Extrados. That is, given the nature or form of an arch, to find the nature of the line forming the top of the superincumbent wall, by whose pressure the arch is kept in equilibrium.*

*Solution.*

LET D be the vertex of the given curve ABCD, and K that of the required line GHK. Put  $a = DK$ ,  $x = AP$  the abscissa,  $y = PC$  the ordinate,  $z = DC$  the arch, and  $R =$  the radius of curvature at the point C.

Now, by the last prop. CI is as  $\frac{\text{sec. } \angle KCP^3}{R}$ .

But, by similar triangles, as  $y : z :: 1$  (radius)  
 $z : \frac{z^3}{y} = \text{sec. } \angle KCP$ ; therefore CI is as  $\frac{z^3}{Ry^3}$ .

Again, in every curve whose ordinate is referred to an axis, the radius of curvature  $R$  is  $= \frac{z^3}{yx - xy}$ ;

wherefore CI will be as  $\frac{yx - xy}{y^3}$ , or CI =  $\frac{yx - xy}{y^3} \times Q$ ; where  $Q$  is a constant quantity  
 whose

Sect. II. *Of the Arches.* 19

whose value will be determined by taking the expression for the given perpendicular DK at the vertex of the curve.

*Corollary.*

HENCE then, as either  $x$  or  $y$  may be supposed to flow uniformly, and consequently either of their second fluxions equal to nothing, by striking either of the terms out of the numerator of the above value of CI, and then exterminating either of the unknown quantities by the equation of the curve, the value of CI will be obtained; as is done in the following examples.

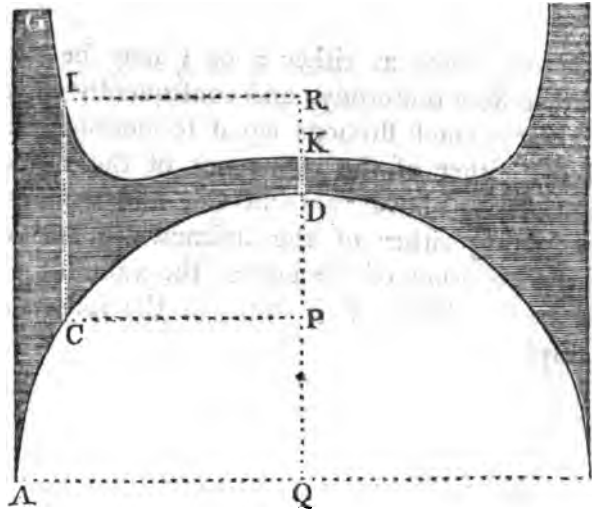
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Ex.



## EXAMPLE I.

*To find the extrados of a circular arch.*



LET  $Q$  be the center and  $D$  the vertex of the given circular arch,  $K$  the vertex of the extrados, and the other lines as in the figure.

Put  $a = DK$ ,  $r = AQ = QD =$  the radius,  $x = DP$ , and  $y = PC = RI$ ,

$$\text{Then } y = \sqrt{2rx - xx}, \dot{y} = \frac{r - x}{\sqrt{2rx - xx}} \times \dot{x},$$

$$\text{and } \ddot{y} = \frac{-r^2 \dot{x}^2}{2rx - xx}^{\frac{3}{2}}, \text{ by making } \ddot{x} = 0. \text{ Hence}$$

CI

$$\begin{aligned}
 \text{CI} &= \frac{y\ddot{x} - x\ddot{y}}{y^3} \times Q \text{ is } = \frac{-x\ddot{y}}{y^3} \times Q = \frac{r^2 \dot{x}^3}{2rx - x\dot{x}^2} \\
 &\times \frac{2rr - x\dot{x}^2}{(r - x)^3 \times \dot{x}^3} \times Q = \frac{r^2 Q}{r - x}. \text{ But, at the vertex} \\
 &x \text{ is } = 0, \text{ and then CI is } = \text{DR} = a = \frac{r^2 Q}{r - 0} \\
 &= \frac{r^2 Q}{r} = \frac{Q}{r}. \text{ Consequently the value of } Q \text{ is} \\
 &= ar. \text{ And the general value of CI or } \frac{r^2 Q}{r - x} \\
 &\text{is } a \times \frac{r}{r - x} = \frac{\text{DK} \times \text{DQ}}{\text{PQ}}.
 \end{aligned}$$

*Otherwise,*

By making  $y$  constant.

THE notation remaining as before: we have  
 $x = r - \sqrt{r^2 - y^2}$ ,  $\dot{x} = \frac{yy}{\sqrt{r^2 - y^2}}$ , and  $\ddot{x} = \frac{r^2 y^2}{rr - yy^{\frac{3}{2}}}$ . Hence CI or  $\frac{y\ddot{x} - x\ddot{y}}{y^3} \times Q$  becomes  
 $\frac{\ddot{x}}{y^2} \times Q = \frac{r^2 Q}{rr - yy^{\frac{3}{2}}}$ . This when  $y = 0$ , gives  
 $a = \frac{Q}{r}$ , and  $Q = ar$  as before. And consequently

quently CI or  $\frac{r^2 Q}{rr - yy^{\frac{1}{2}}}$  is  $= a \times \frac{r}{\sqrt{r^2 - y^2}}$   
 $= \frac{DK \times DQ'}{PQ'}$  as before.

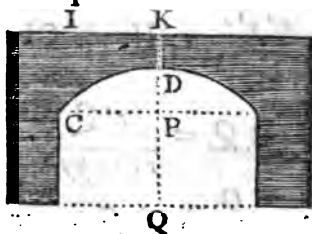
Hence the equation to the curve KI is  $v =$   
 $(KR = ax + x - IC =) a + x - \frac{ar^3}{r - x}$  or  $=$   
 $a + r - \sqrt{r^2 - y^2} - \frac{ar^3}{rr - yy^{\frac{1}{2}}}$

*Corollary 1.*

HENCE KIG is a curve running up an infinite height towards G, the perpendicular AG being an asymptote to it: And the curve is accurately as represented in the figure, when the thickness DK at the top is 1-15th of the span,

*Corollary 2.*

BUT the curve KIG is quite inconvenient for the form of the extrados of any bridge; however a freight horizontal line IK might be used instead of it, if the materials of which the arch is built, could



be

be so chosen, as that they might increase in their specific gravity from DK towards CI, continually as the cube of the secant of the arch from D. And this again perhaps would be quite impracticable: But if a circular arch and a right line at the top were *necessarily* required, the proportion of DK to the radius DQ may be found so as the arch may be *nearly* in equilibrium thus:

When KI is a right line, then KR in the figure to the example, must be nothing; or rather when the curve crosses the horizontal line, then KR is equal to nothing; put its value then, as found above, equal to 0, and we shall

have  $\frac{ar^3}{rr-yy} - a - r + \sqrt{r^2-y^2} = 0$ , and

from this equation, by assuming one of the quantities,  $a$ ,  $y$ , the corresponding value of the other may be found for the point where the curve crosses the horizontal line; so from hence

the general value of  $a$  is  $\left( \frac{r - \sqrt{r^2 - y^2}}{r^3 - \sqrt{r^2 - y^2}^3} \times \right.$

$$\left. \frac{rr-yy}{rr-yy} \right)^{\frac{1}{3}} = \frac{(rr-yy)^{\frac{1}{3}}}{r^2 + r\sqrt{r^2-y^2} + r^2 - y^2} =$$

$$\frac{PQ^3 = v^3}{r^2 + rv + v^2} = \frac{(r-x)^3}{3r^2 - 3rx + x^2}. \text{ Now this va-}$$

lue of  $a$  or DK evidently becomes = 0 when the arch consists of the whole semi-circle; but  
when

when the arch is less than the semicircle,  $a$  will have a finite value, and between 60 and 120 degrees many arches of equilibration of a certain thickness at top may be found. Thus, if the half arch DC contain 30 degrees; then its sine  $y$  or PC is  $= \frac{1}{2}r$ ; which being substituted for it in the above general value of  $a$ , we have

$$a = \frac{7\sqrt{3}-6}{37} \times \frac{1}{2}r, \text{ or } = \frac{1}{4}r \text{ extremely near;}$$

that is, DK is  $= \frac{1}{4}$  of DQ or  $\frac{1}{4}$  of 2PC the span when the curve cuts the horizontal line directly above the point in the circle which answers to 30 degrees. And if DC were an arch

$$\text{of 45 degrees; then } y = r\sqrt{\frac{1}{2}}, \text{ and } a = \frac{3\sqrt{2}-2}{14}$$

$$\times r = \frac{16r}{100}, \text{ or } \frac{1}{6} \text{ of the span nearly. Also, if}$$

$$\text{DC were 60 degrees; then } y = r\sqrt{\frac{1}{4}}, \text{ and } a =$$

$$\frac{1}{14} \text{th of } r = \frac{7r}{100}, \text{ or } \frac{1}{14} \text{ of the span nearly.——}$$

So that in each of these cases the points C and D would be in equilibrium; but then about the middle parts between D and C, or rather nearer to D than to C, the materials should be a little lighter than at D and C, and the exact proportion in which their gravity should be diminished, might easily be found by calculation; so in the first case, in particular, the specific gravity of the materials in the middle of the arch between D and C, that is at 15 degrees from D, should be to that at D or C, as 278 to 284, which is  
but



axe horizontal; and let  $b$  denote the horizontal semi-axe  $AQ$ , and  $r$  the vertical one  $DQ$ , and all the other letters as in the last example.

Then, by the nature of the ellipse,  $r : b :: \sqrt{2rx - xx} : y = \frac{b}{r} \sqrt{2rx - xx}$ ; hence  $\dot{y} = \frac{b\dot{x}}{r} \times \frac{r-x}{\sqrt{2rx - xx}}$ , and  $\ddot{y} = \frac{-br\dot{x}^2}{2rx - xx)^{\frac{3}{2}}}$  by

making  $\dot{x}$  constant. Then  $CI = \frac{-\dot{x}\ddot{y}}{y^3} \times \mathcal{Q}$  is  

$$= \frac{br\dot{x}^3 \mathcal{Q}}{2rx - xx)^{\frac{3}{2}}} \times \frac{r^3}{b^3 \dot{x}^3} \times \frac{2rx - xx)^{\frac{1}{2}}}{(r-x)^3} = \frac{r^4 \mathcal{Q}}{b^3 (r-x)^3}.$$

But when  $x$  is  $= 0$ , this expression becomes  $a = \frac{r\mathcal{Q}}{bh}$ , and then  $\mathcal{Q} = \frac{abh}{r}$ ; consequently  $CI$  is  

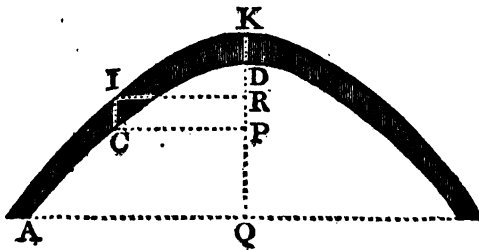
$$= a \times \left[ \frac{r}{r-x} \right]^3 = \frac{DR \times DQ^3}{PQ^3},$$
 the same as in the circle.—And the same expression may be brought out by making  $\dot{y}$  constant.

Hence the nature of the curve  $KI$  is thus expressed,  $KR = a + x - a \times \left[ \frac{r}{r-x} \right]^3 = a + r - \frac{r}{b} \sqrt{bh - yy} - \frac{ab^3}{bb - yy)^{\frac{3}{2}}}$ , and is of the same kind with that in the last example.—But the elliptic arch may take a streight line at top better than the circular one, when the longer axe is hori-

horizontal, because the arch is flatter, or of a less curvature; and worse than the circular arch, when the shorter axe is horizontal.

## EXAMPLE 3.

*To determine the figure of the extrados of a parabolic arch of equilibration.*



Put  $a = KD$ ,  $r = DQ$ ,  $h = QA$ ,  $x = DP$ , and  $y = PC = RI$ .

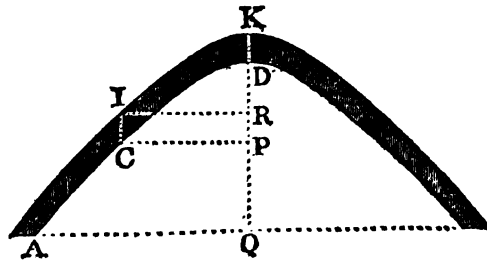
Then, by the nature of the curve,  $bb : yy :: r : x = \frac{ryy}{bb}$ ; and hence  $\ddot{x} = \frac{2ry\dot{y}}{bb}$ , and  $\ddot{x} = \frac{2r\ddot{y}}{bb}$ , by making  $y$  constant. Then  $CI = \frac{\ddot{x}}{y^2} \times Q$  is  $= \frac{2rQ}{bb} =$  a constant quantity  $= a$ ; that is,  $CI$  is every-where equal to  $KD$ .



Consequently  $KR$  is  $= DP$ ; and since  $RI$  is  $= PC$ , it is evident that  $KI$  is the same parabolic curve with  $DC$ , and may be placed any height above it.

#### EXAMPLE 4.

*To find the figure of the extrados for an hyperbolic arch of equilibration.*



Put  $a = KD$ ,  $r =$  the semi-transverse, and  $b =$  the semi-conjugate axe,  $x = DP$ , and  $y = PC = RI$ .

Then, by the nature of the hyperbola,  $y = \frac{b}{r} \sqrt{2rx + xx}$ ; hence  $\dot{y} = \frac{bx}{r} \times \frac{r+x}{\sqrt{2rx + xx}}$ , and  $\ddot{y} = \frac{-bx^2}{(2rx + xx)^{3/2}}$ , by making  $x$  constant.

Wherefore  $CI$  or  $\frac{-x\ddot{y}}{y^3} \times \mathcal{Q} = \frac{r^2 \mathcal{Q}}{b^2 x r + x^2}$ . But when

when  $x = 0$ , this expression becomes  $\frac{rQ}{bh} = a$ ;

hence  $Q = \frac{abh}{r}$ , and consequently CI or

$$\frac{r^2 Q}{b^2 \times r + x^2} \text{ is } = \frac{ar^2}{r + x^2}.$$

Whence the equation to the curve KI required will be  $KR = (a + x - CI) = a + x$

$$- \frac{ar^2}{r + x^2} = a - r + \frac{r}{b} \sqrt{bb + yy} - \frac{ab^2}{bb + yy^{\frac{3}{2}}}.$$

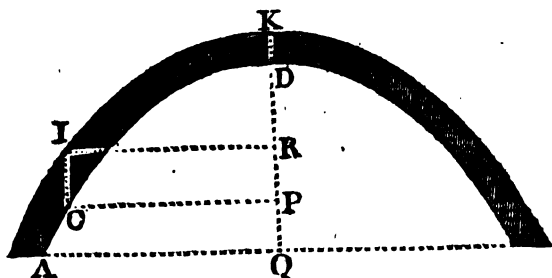
*Scholium,*

In this hyperbolic arch then, it is evident that the extrados KI continually approaches nearer to the intrados; whereas in the circular and elliptic arches, it goes off continually farther from it; and in the parabola, the two curves keep always at the same distance; observing however that by the distance between the two curves, in each of these cases, is meant their distance in the vertical direction.

Ex-

## EXAMPLE 5.

To find the extrados for a catenarian arch of equilibration.



LET  $a = KD$ ,  $x = DP$ , and  $y = PC = RI$ , as before; also let  $c$  denote the constant tension of the curve at the vertex.

Then, by the nature of the catenary,  $y$  is  $= c \times \text{hyp. log. of } \frac{c + x + \sqrt{2cx + xx}}{c}$ ; hence,

taking the fluxions, we have  $\dot{y} = \frac{c\dot{x}}{\sqrt{2cx + xx}}$ ,

and  $\ddot{y} = -c\dot{x}^2 \times \frac{c + x}{2cx + xx\sqrt{2cx + xx}}$ , by making  $\dot{x}$

constant. Wherefore  $CI$  or  $\frac{-\ddot{y}}{\dot{y}^2} \times \mathcal{Q}$  is  $= \frac{c + x}{cc} \times \mathcal{Q}$ . But at the vertex  $x$  is  $= 0$ , and  $CI = a = \frac{\mathcal{Q}}{c}$ ; consequently  $\mathcal{Q}$  is  $= ac$ . This being written

written for it, there results  $CI = \frac{c+x}{c} \times a =$   
 $a + \frac{ax}{c}.$

Hence, for the nature of the curve KI, we  
 have  $KR = (a+x-CI) = x - \frac{ax}{c} = \frac{c-a}{c} \times x.$

*Corollary.*

AND hence the abscissa DP is to the abscissa KR, always in the constant proportion of  $c$  to  $c-a$ . So that, when  $a$  is less than  $c$ , R and the curve KI lies below the horizontal line; but when  $a$  is greater than  $c$ , they lie above it; and when  $a$  is equal to  $c$ , KR is always equal to nothing, and KI or the extrados coincides with the horizontal line.

As  $a$  diminishes, the line KI approaches nearer to DC in all its parts, till when  $a$  entirely vanishes, or is so little in respect of  $c$  as to be omitted in the expression  $\frac{c-a}{c} \times x = KR$ , the two curves quite coincide throughout.

*Scholium.*

As we have found above that the extrados will be a straight horizontal line when  $a$  is equal  
 to

### 32 *The PRINCIPLES of BRIDGES.*

to  $c$ , I shall here make a calculation to determine, in that case, the value of  $c$ , and consequently of  $a$  with respect to  $x$  and  $y$ , or a given span and height of an arch.

Now the equation to the curve expressed in terms of  $c$ ,  $x$ , and  $y$ , is  $y = c \times \text{hyp. log. of } \frac{c+x+\sqrt{2cx+xx}}{c}$ ; and when  $x$  and  $y$  are given, the value of  $c$  may be found from this equation, by the method of trial and error. But as the process would be at best but a tedious one, and perhaps the method not easy in this case to be practised by every person, I shall here investigate a series for finding the value of  $c$  from those of  $x$  and  $y$  in a direct manner.

Since then  $y$  is  $= c \times \text{hyp. log. of } \frac{c+x+\sqrt{2cx+xx}}{c}$ , by taking the fluxion of

this equation, we have  $\dot{y} = \frac{c\dot{x}}{\sqrt{2cx+xx}} =$

$\frac{\frac{1}{2}d\dot{x}}{\sqrt{dx+xx}}$  by writing  $d$  for  $2c$ ; and by expanding this expression into a series, it becomes

$$\dot{y} = \frac{1}{2}\dot{x}\sqrt{\frac{d}{x}} \times$$

$$: 1 - \frac{x}{2d} + \frac{1.3x^2}{2.4d^2} - \frac{1.3.5x^3}{2.4.6d^3} + \frac{1.3.5.7x^4}{2.4.6.8d^4} \&c.$$

and,

and, by taking the fluents we have  $y = \sqrt{dx} \times$

$$: 1 - \frac{x}{2.3d} + \frac{1.3x^2}{2.4.5d^2} - \frac{1.3.5x^3}{2.4.6.7d^3} + \frac{1.3.5.7x^4}{2.4.6.8.9d^4} \&c.$$

and hence, by dividing by  $x$ , we have  $\frac{y}{x} = \sqrt{\frac{d}{x}} \times$

$$: 1 - \frac{x}{2.3d} + \frac{1.3x^2}{2.4.5d^2} - \frac{1.3.5x^3}{2.4.6.7d^3} + \frac{1.3.5.7x^4}{2.4.6.8.9d^4} \&c.$$

or, by writing  $v$  for  $\frac{y}{x}$  and  $w$  for  $\sqrt{\frac{d}{x}}$ , it is  $v =$

$$w - \frac{1}{2.3w} + \frac{1.3}{2.4.5w^3} - \frac{1.3.5}{2.4.6.7w^5} + \frac{1.3.5.7}{2.4.6.8.9w^7} \&c.$$

Then, by reverting this series, we have  $w =$

$$v + \frac{1}{6v} - \frac{37}{360v^3} + \frac{547}{5040v^5} - \frac{337}{5600v^7} \&c. \text{ And}$$

hence, by squaring, &c. and restoring the original letters, it is ( $\frac{1}{2}d = \frac{1}{2}xw^2 =$ )  $c = \frac{1}{2}x \times$

$$: \frac{y^2}{x^2} + \frac{1}{3} - \frac{8x^2}{45y^2} + \frac{691x^4}{3780y^4} - \frac{23851x^6}{453600y^6} \&c.$$

where a few of the first terms are sufficient to determine the value of  $c$  pretty nearly.

Now, for an example in numbers, suppose the height of the arch to be 40 feet, and its span 100, which are nearly the dimensions of the middle arch of Blackfriar's Bridge at London. Then  $x = 40$ , and  $y = 50$ ; which being substituted for them in this series, it gives  $c = 36.88$  feet nearly. So that to have made that arch a catenarian one, with a straight line above, the top of the arch must have been almost of the

F

immense

immense thickness of 37 feet, to have kept it in equilibrium.

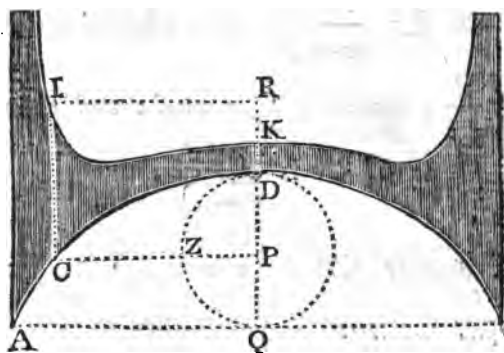
But if the height and span be 40 and 100 feet, as above, and the thickness of the arch at top be assumed equal to 6 feet, then the extrados will not be a right line, but as it is drawn in the figure to this example, which figure is accurately constructed according to these dimensions.

It may be farther remarked, that the curves in these last three examples, viz. the parabola, hyperbola, and catenary, are all very improper for the arches of a bridge consisting of several arches; because it is evident from their figures, which are all accurately constructed, that all the building or filling up of the flanks of the arches will tend to destroy the equilibrium of them. But in a bridge of one single arch whose extrados rises pretty much from the spring to the top, one of these figures will answer better than any of the former ones.

Ex-

## EXAMPLE 6.

To determine the extrados of a cycloidal arch of equilibration.



LET  $DZQ$  be the circle from which the cycloid  $DCA$  is generated, and the other lines as before.

Put  $a = DK$ ,  $x = DP$ , and  $y = PC = RI$ ; also put  $d = DQ$  the diameter of the circle, and  $z =$  the circular arc  $DZ$ .

Then, by the nature of the cycloid,  $CZ$  is always equal to  $DZ = z$ ; and, by the nature of the circle,  $PZ$  is  $= \sqrt{dx - xx}$ ; wherefore  $PC$  or  $y = (CZ + ZP) = z + \sqrt{dx - xx}$ . Hence

$$\dot{y} = \dot{z} + \frac{\frac{1}{2}d - x}{\sqrt{dx - xx}} \times \dot{x}; \text{ but } \dot{z} = \frac{\frac{1}{2}d\dot{x}}{\sqrt{dx - xx}} \quad \text{by}$$

F 2



by the nature of the circle; therefore  $\dot{y} = \frac{d-x}{\sqrt{dx-xx}} \times \dot{x} = \dot{x} \sqrt{\frac{d-x}{x}}$ ; and then  $\ddot{y} = \frac{-d\dot{x}^2}{2x\sqrt{dx-xx}}$ , making  $\dot{x}$  constant. Hence CI  $= \frac{-x\ddot{y}}{y'} \times \mathcal{Q} = \frac{\frac{1}{2}d\mathcal{Q}}{d-x}$ . But when  $x=0$ , CI is  $= a = \frac{\mathcal{Q}}{2d}$ ; therefore  $\mathcal{Q} = 2ad$ ; and then the general value of CI is  $\frac{add}{d-x}$ .

Consequently KR  $= (a + x - \text{CI}) = a + x - \frac{add}{d-x}$  will express the nature of the curve

KI; which resembles that for the circle and ellipse, as evidently appears by comparing the figures together, each of them being accurately constructed. But this figure seems to be rather better than either of them, as the extrados approaches rather nearer to a right line, and extends farther out before it is bent upwards.

Other examples of known curves might be given; but those that have been put down already, seem to be the fittest for real practice; and there is a sufficient variety among them, to suit the various circumstances of convenience, strength, and beauty.

I shall

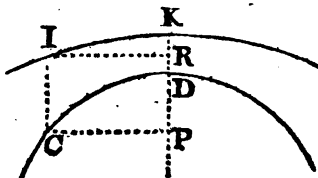
I shall now proceed to another general problem, which is the reverse of the last one, and determines the figure of the intrados for any given figure of the extrados, so that the arch may be in equilibrium in all its parts.

### PROPOSITION V.

*HAVING the Extrados given, to find the Intrados. That is, having given the nature or form of a line bounding the top of a wall above an arch; to find the figure of the arch, so that by the pressure of the superincumbent wall, the whole may remain in equilibrium.*

*Solution.*

Put  $a = DK$  the thickness of the arch at top,  $x = DP$  the abscissa of the intrados  $DC$ ,  $z = KR$  the abscissa of the given extrados  $KI$ , and  $y = PC = RI$  their equal ordinates.



Then, by the last proposition,  $CI$  is  $= \frac{y''x - x''y}{y'}$

$\times 2$ ; but  $CI$  is also evidently equal to  $a + x - z$ ; therefore

therefore  $a + x - z$  is  $= \frac{\dot{y}\ddot{x} - x\ddot{y}}{\dot{y}^2} \times \mathcal{Q} = \frac{\mathcal{Q}}{\dot{y}} \times$

the fluxion of  $\frac{x}{y}$ ; where  $\mathcal{Q}$  is a constant quan-

tity, as used in the last proposition, and always to be determined from the nature or conditions of each particular case.

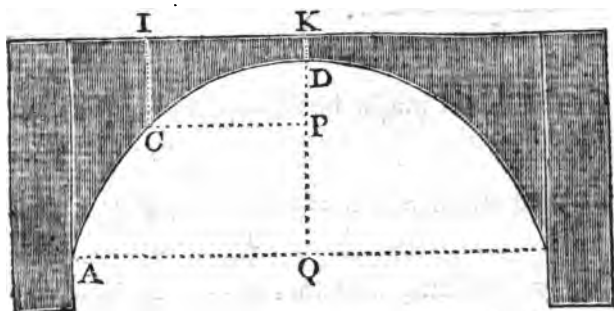
Hence then, by substituting in this equation the given value of  $z$  instead of it, as expressed in terms of  $y$ , the resulting equation will then involve only  $x$  and  $y$  together with their first and second fluxions, besides constant quantities. And from it the relation between  $x$  and  $y$  themselves may be found, by the application of such methods as may seem to be best adapted to the particular form of the given equation to the extrados. In general, a proper series for the value of  $x$  in terms of  $y$  is to be assumed with indeterminate coefficients; which series being put into fluxions, striking out of every term the fluxion of  $y$ ; and the result fluxed again, striking out from every term of this also the fluxion of  $y$ ; the last expression drawn into  $\mathcal{Q}$  being equated to  $a + x - z$ , there will be produced an equation from which will be found the values of the coefficients of the terms in the assumed value of  $x$ .

But in the particular case when  $z$  is always nothing, or the extrados a right horizontal line,  
a dif-

a different and easier process obtains, as in this following example.

## EXAMPLE.

To find an arch of equilibration whose extrados shall be a horizontal line.



Making the notation as in the proposition, we have  $z = 0$ , and therefore  $a + x = \frac{Q}{y} \times$  the fluxion of  $\frac{x}{y}$ .

Now assume  $y = \frac{x}{v}$ ; then  $\frac{x}{y} = v$ , and  $\frac{Q}{y} \times$  flux. of  $\frac{x}{y} = \frac{Qv\dot{v}}{x}$ ; that is,  $a + x = \frac{Qv\dot{v}}{x}$ ; hence  $ax + x\dot{x} = Qv\dot{v}$ . Then, by taking the fluents, we have  $2ax + x^2 = Qv^2$ ; hence  $v = \sqrt{\frac{2ax + x^2}{Q}}$ ,  
and

and consequently  $\dot{y} = \left(\frac{\dot{x}}{v}\right) = \frac{\mathcal{Q}^{\frac{1}{2}} \dot{x}}{\sqrt{2ax + xx}}$ . Then

the fluent of this is  $y = \mathcal{Q}^{\frac{1}{2}} \times \text{hyp. log. of } 2a + 2x + 2\sqrt{2ax + xx}$ ; but when  $x = 0$ , this is  $\mathcal{Q}^{\frac{1}{2}} \times \text{hyp. log. of } 2a$ ; therefore the correct fluent is  $y = \mathcal{Q}^{\frac{1}{2}} \times \text{hyp. log. of } \frac{a + x + \sqrt{2ax + xx}}{a}$ .

Or the fluent might be otherwise found thus.

THE equation  $a + x = \frac{\dot{y}\ddot{x} - \dot{x}\ddot{y}}{\dot{y}^3} \times \mathcal{Q}$ , sup-

posing  $\dot{y}$  constant, becomes  $a + x = \frac{\mathcal{Q}\ddot{x}}{\dot{y}^2}$ , or

$a\dot{y}^2 + x\dot{y}^2 = \mathcal{Q}\ddot{x}$ ; multiply by  $\dot{x}$ , and then

$a x \dot{y}^2 + x x \dot{y}^2 = \mathcal{Q} \dot{x} \ddot{x}$ ; and hence, by taking the

fluents,  $2ax\dot{y}^2 + x^2\dot{y}^2 = \mathcal{Q}\dot{x}^2$ ; consequently

$\dot{y}^2 = \frac{\mathcal{Q}\dot{x}^2}{2ax + xx}$ , or  $\dot{y} = \frac{\mathcal{Q}^{\frac{1}{2}} \dot{x}}{\sqrt{2ax + xx}}$ . And then

the rest will be as above.

Now the value of  $\mathcal{Q}$  will be found by writing in this equation some particular correspondent known values of  $x$  and  $y$ : thus when P arrives at Q, then  $x = DQ = r$ , and  $y = QA = h$ ; these being substituted for them, we have  $h = \mathcal{Q}^{\frac{1}{2}} \times \text{hyp.}$

hyp. log. of  $\frac{a+r+\sqrt{2ar+rr}}{a}$ , and consequently  $\mathcal{Q}^{\frac{1}{2}} = \frac{b}{\text{hyp. log. of } a+r+\frac{\sqrt{2ar+rr}}{a}}$ .

Wherefore the general value of  $y$  is thus,  $y =$

$$b \times \frac{\text{hyp. log. } \frac{a+x+\sqrt{2ax+xx}}{a}}{\text{hyp. log. } \frac{a+r+\sqrt{2ar+rr}}{a}}.$$

Hence, when  $\mathcal{Q}^{\frac{1}{2}}$  is  $= a$ , the curve DC is the catenary; and in general the ordinate is everywhere to the corresponding ordinate of the catenary whose tension at the vertex is  $a$ , as  $b$  is to  $a \times \text{hyp. log. of } \frac{a+r+\sqrt{2ar+rr}}{a}$ .

If  $x$  were desired in terms of  $y$ , it would be thus. Put  $A =$  the hyp. log. of  $a$ , and  $D = \frac{1}{b} \times \text{hyp. log. of } \frac{a+r+\sqrt{2ar+rr}}{a}$ ; then

$$Dy + A = \text{hyp. log. of } a+x+\sqrt{2ax+xx}:$$

Again, put  $N =$  the number whose hyp. log. is  $Dy + A$ ; then  $N = a+x+\sqrt{2ax+xx}$ ; and

$$\text{hence } x = \frac{(N-a)^2}{2N}, \text{ or } a+x = KP = \frac{N^2+a^2}{2N}.$$

By taking  $AQ = b = 50$ , and  $DQ = r = 40$ , also  $DK = a = 6$ . Then the hyp. log.

G

of

of  $\frac{a+r+\sqrt{2ar+rr}}{a}$  is = the hyp. log. of

$\frac{46+4\sqrt{130}}{6}$  = the hyp. log. of  $15.26784 =$

$2.7257487$ ; by which dividing  $b = 50$ , the quotient is  $18.343584$ . So that the ordinate  $y$  will be constantly in that case equal to  $18.343584$

$\times$  the hyp. log. of  $\frac{6+x+\sqrt{12x+xx}}{6}$ . Also

$\frac{1}{18.343584} = .05451497$  is =  $D$ , and  $A =$  hyp.

log. of  $6 = 1.7917594$ ; then  $N =$  the number whose hyp. log. is  $1.7917594 + .05451497$ . And then by assuming several values of one of the letters  $x, y$ , the corresponding values of the other will be found from one of the two equations above.

And in this manner were calculated the numbers in the following table; from which the curve being constructed, it will be as appears in the figure to the example.—And thus we have an arch in equilibrium in all its parts, and its top a streight line, as is generally required in most bridges; or at least they are so near a horizontal line, that their difference from it will cause no sensible ill consequence. It is also both both of a graceful figure, and of a convenient form for the passage through it. So that there can be no good reason for neglecting to use it in works of any consequence.

*The*

*The Table for Constructing the Curve in this Example.*

Value of KI	Value of IC	Val. of KI	Value of IC	Val of KI	Value of IC
0	6.000	21	10.381	36	21.774
2	6.035	22	10.858	37	22.948
4	6.144	23	11.368	38	24.190
6	6.324	24	11.911	39	25.505
8	6.580	25	12.489	40	26.894
10	6.914	26	13.106	41	28.364
12	7.330	27	13.761	42	29.919
13	7.571	28	14.457	43	31.563
14	7.834	29	15.196	44	33.299
15	8.120	30	15.980	45	35.135
16	8.430	31	16.811	46	37.075
17	8.766	32	17.693	47	39.126
18	9.168	33	18.627	48	41.293
19	9.517	34	19.617	49	43.581
20	9.934	35	20.665	50	46.000

The above numbers may be feet or any other lengths of which DQ is 40 and QA is 50. But when DQ is to QA in any other proportion than that of 4 to 5, or when DK is not to DQ as 6 to 40 or 3 to 20; then the above numbers will not answer; but others must be found by the same rule, to construct the curve by.

In the beginning of the table, as far as 12, the value of KI is made to differ by 2, because the



the value of IC in that part increases so very slowly.

Other examples of given extrados might be taken ; but as there can scarcely ever be any real occasion for them, and as the trouble of calculation would be, in most cases, extremely great, they are omitted.

SEC-

## SECTION III.

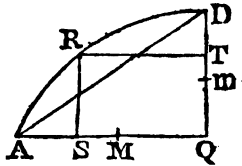
## Of the Piers.

## PROPOSITION VI.

To find the distance  $QM$  of the center of gravity of the given circular arc  $AD$ , from  $DQ$  the versed sine of the said arc,  $QA$  being its right sine.

*Solution.*

Put  $r$  = the radius,  $z$  = any arc  $DR$ , and  $x$  = its sine  $TR$  or  $QS$ .



Then, by mechanics, the force of a particle  $z$  of the curve placed at  $R$  is  $TR \times z = xz$ ; and the force of all the particles will be equal to the fluent of  $xz$ ; which must be equal to  $QM$  drawn into the whole line; that is,  $QM \times z$  = the fluent of  $xz$ , or  $QM = \frac{1}{z} \times$  fluent of  $xz$ . And this is a general theorem, whether  $z$  be a line, surface, or solid; supposing the two former to be affected with gravity.

Now,

Now, by the nature of the circle,  $\dot{z} = \frac{rx}{\sqrt{rr - xx}}$ ; and therefore  $x\dot{z} = \frac{rxx}{\sqrt{rr - xx}}$ ; the correct fluent of which is  $r \times r - \sqrt{rr - xx}$ . Consequently QM is  $= r \times \frac{r - \sqrt{rr - xx}}{z}$ ; which, when  $x = QA$ , and  $z =$  the arc AD, becomes  $QM = r \times \frac{r - \sqrt{r^2 - QA^2}}{ARD} =$  the distance from DQ required.

Or, since  $r - \sqrt{r^2 - QA^2}$  is  $= QD$ , the same distance QM will be expressed by  $r \times \frac{DQ}{ARD}$ .

Or, lastly, since  $r \times QD$  is half the square of the chord AD, the same distance QM will be equal to  $\frac{AD^2}{2ARD}$  or  $\frac{AQ^2 + QD^2}{2ARD}$ .

*Corollary.*

When ARD is a quadrant, then  $AQ = QD = r$ , and the rule is  $QM = \left( \frac{rr}{ARD} = \frac{rr}{.7854 \times 2r} \right. \\ \left. = \right) \frac{r}{1.5708}$ . Or  $QM = .637r$  nearly, or  $= \frac{7}{11}r$  extremely near.

[PROPO-

## PROPOSITION VII.

THE figure being the same as in the last proposition, in is required to find the distance Qm of the center of gravity of the arc ARD from the sine AQ.

*Solution.*

As in the last proposition, QM will be =  $\frac{I}{AR}$   $\times$  the fluent of SR  $\times$  AR.

But, putting  $z = AR$ ,  $x = QS = RT$ ,  $r$  = the radius,  $b = DQ$ , and  $s = QA$ , we shall have

$$z = AR = \frac{-rx}{\sqrt{rr - xx}}, \text{ and } SR = b - r +$$

$$\sqrt{rr - xx}; \text{ hence } SR \times AR = \frac{r - b}{\sqrt{rr - xx}} \times$$

$rx - rx = \overline{b - r} \cdot z - rx$ ; the correct fluent of which is  $\overline{b - r} \cdot z + \overline{s - x} \cdot r$ .

Consequently Qm is =  $b - r + \frac{s - x}{z} \cdot r =$   
 $b - r + \frac{AS}{z} \cdot r$  And when R arrives at D, it  
 is Qm =  $b - r + \frac{sr}{A}$ .

Or,

Or, since  $r$  is  $= \frac{ss + hh}{2b}$ , the same distance  
 $Qm$  will be  $= \frac{hb - ss}{2b} + \frac{hb + ss}{2b} \cdot \frac{s}{A}$ ; where  
 $A$  is the whole arc  $ARD$ .

*Corollary.*

When  $ARD$  is a quadrant, then  $b$  and  $s$  are  
 each  $= r$ , and the rule is  $\frac{rr}{A}$ , the same as in the  
 corollary to the last.

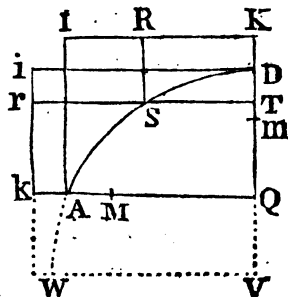
## PROPOSITION VIII.

To find the distance  $QM$  of the center of gravity of the space  $AIKDSA$  from  $KQ$ ; supposing  $DA$  to be a circular arc whose sine is  $AQ$ , its versed sine  $QD$ , and  $AI$ ,  $IK$ , parallel to  $DQ$ ,  $QA$  respectively.

*Solution.*

Draw  $RS$ ,  $ST$  parallel to  $DQ$ ,  $QA$ . And put  $a = DK$ ,  $r = VD = VW$  the radius of the circle,  $x = TS = KR$ , and  $z =$  the area  $DSRK$ .

Then, as in prop. 6, we shall have  $QM = \frac{1}{z} \times$  the fluent of  $xz$ .



But  $\dot{z}$  is  $= RS \times \dot{x}$ , and  $RS = KD + DT = a + r - \sqrt{rr - xx}$ . Consequently  $x\dot{z}$  is  $= RS \times \dot{x} = a\dot{x}x + r\dot{x}x - \dot{x}x\sqrt{rr - xx}$ ; the correct fluent of which is  $\frac{a+r}{2} \cdot x^2 - \frac{r^2 - rr - xx}{3}^{\frac{1}{2}}$ .

H

Wherefore

Wherefore QM is =  $\frac{a+r}{2z} \cdot x^2 - \frac{r^2 - \sqrt{rr - xx}}{3z}$

$$= \left( \frac{a+r}{2} \cdot \frac{xx}{z} - \frac{rxx + r - \sqrt{rr - xx} \cdot rr - xx}{3z} \right)$$

$$= \frac{3a+r}{6} \cdot \frac{xx}{z} - \frac{TD}{3} \cdot \frac{r - TD}{z} =$$

$$\frac{3a+r}{6z} \cdot TS^2 - 2TD \cdot \frac{r - TD}{z} \text{ or } = \frac{r^2 - y^2}{2z} \cdot m$$

—  $\frac{r^2 - y^2}{3z}$ , putting  $m = VK$  and  $y = VT$ .

And when SR arrives at AI, then QM is

$$= \frac{3a+r}{6AIKDSA} \cdot QA^2 - 2QD \cdot \frac{r - QD}{z} = \frac{r^2 - VQ^2}{2A} \cdot VK$$

—  $\frac{r^2 - VQ^2}{3A}$ ; putting  $A$  for the whole space AIKDW.

## Corollary 1.

WHEN DA is a quadrant; then the space AIKDSA or AIKQ—ASDQ is =  $a + r \cdot r - .7854rr = a - .2146r \times r$ , and  $QA = QD = r$ .

Wherefore, in that case, QM =  $\frac{3a+r}{a + .2146r} \times \frac{1}{2}r$

$$= \frac{3a+r}{3a + .6438r} \times \frac{1}{2}r.$$

Or QM is =  $\frac{3a+r}{3a + \frac{1}{3}r} \times \frac{1}{2}r = \frac{9a+3r}{9a+2r} \times \frac{1}{2}r$   
nearly.

nearly. Or, rather, it is  $= \frac{3a+r}{3a+\frac{2}{14}r} \times \frac{1}{2}r =$   
 $\frac{42a+14r}{42a+9r} \times \frac{1}{2}r$  extremely near.

## Corollary 2.

WHEN  $a$  is nothing, then (AD being a quadrant) QM is  $= \frac{r}{1.2876}$ . Or it is  $\frac{7}{8}r$  very nearly.

And when  $a = \frac{1}{2}r$ , then QM is  $= \frac{r}{1.4657}$ .  
 Or  $\frac{11}{12}r$  very nearly.

Lastly, when  $a = \frac{1}{14}r$ , which is nearly the proportion in pretty large arches; then QM is  $= \frac{r}{1.406}$ . Or  $\frac{7}{8}r$  very nearly.

## PROPOSITION IX.

*To find the distance of the center of gravity of the space kiDSA from the fine QA of the circular arc ASD; where ki is perpendicular to QAk, and the rest of the lines as in the last figure.*

## Solution.

Put  $a = kA$ ,  $s = AQ$ ,  $m = kQ = a + s$ ,  $r = VW = VD$  the radius,  $z =$  any variable space krSA, and  $x = TS$  the fine of the arc SD, Also  $\Delta =$  the space kiDSA,

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Then



Then  $rS = m - x$ , and, by the circle,  $VT = \sqrt{rr - xx}$ ; hence  $\dot{x} = rS \times VT = \frac{m - x}{\sqrt{rr - xx}}$   
 $x - xx$ ; consequently  $VT \times \dot{x} = \frac{m - x}{2} \times -x\dot{x}$ ;  
 the correct fluent of which is  $\frac{s^3 - x^3}{2} \times m$   
 $- \frac{s^3 - x^3}{3}$ . Wherefore the distance from  $VW$   
 is  $Vm = \frac{s^3 - x^3}{2x} \times m - \frac{s^3 - x^3}{3x}$  for the general  
 space  $krSA$ .

And when  $S$  arrives at  $D$ ,  $x$  is  $= 0$ ; and then  
 $Vm$  is  $= \frac{s^3 m}{2A} - \frac{s^3}{3A} = \frac{3m - 2s}{6A} \times s^3 = \frac{3a + s}{6A}$   
 $\times s^3 = \frac{3kA + AQ^3}{6kiDSA} \times AQ^3 =$  the distance of  
 the center of gravity from  $VW$ .

*Corollary.*

WHEN  $A$  coincides with  $W$ , or the arc a quadrant, then  $s$  is  $= r$ ; and the rule becomes as in Corollary 1 to the last. Also the 2d Corollary to that may be understood here, making the same suppositions as in it.

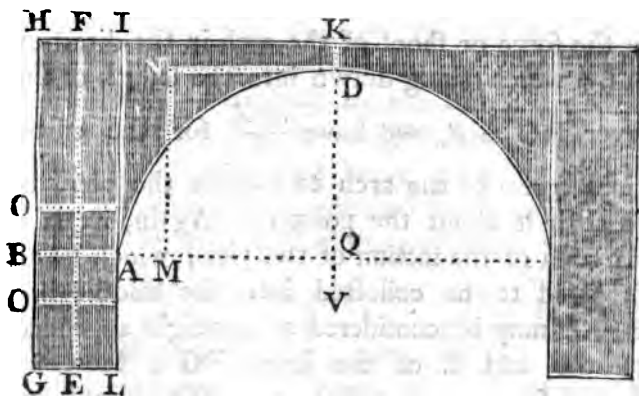
*Scholium.*

THE four preceding propositions are premised as necessary to the examples to the following general one, which determines the thickness of the

the piers necessary to resist the spread or shoot of any given arch, and that whether the whole or part or none of it is immerfed in water. Instances only of circular arcs are here given; because that in determining the drift of the arch, whatever its curve may be, it will make little or no difference by supposing it to be circular.

### PROPOSITION X.

*To find the thicknefs of the piers of an arch, necessary to keep the arch in equilibrium, or to resist its shoot or drift; independent of any other arches.*



### *Solution.*

LET IKDA be the half arch, and IHGL the pier to support it, moveable about the point G, and bisected by the perpendicular EF.

Through

Through the center of gravity of the arch AIKD draw MN perpendicular to AQ the semispan, and meeting DN drawn parallel to AQ in N. And continue QA to meet GH in B.

Put  $a = DK$ ,  $b = DQ = MN$ ,  $c = AM$ ,  $A =$  the area or section AIKD of the arch,  $d = AL = BG$ ,  $e = FE$ , and  $x = AB = GL$  the required breadth of the pier,

Now (by prop. 63 *Emer. Mechan.*) the weight of the arch is to its pressure in the direction AB, as NM is to MA; hence  $b : c :: A : \frac{cA}{b}$  = the force or shoot of the arch in the direction AB; which being drawn into the length of the lever  $BG = d$ , we have  $\frac{cdA}{b}$  for the efficacious force of the arch to overset the pier, or to turn it about the point G. Again,  $ex$  is = the area of the section of the pier; which being supposed to be collected into the middle line EF, it may be considered as a weight appended to the end E of the lever EG; therefore  $ex \times EG = \frac{1}{2}exx$  will be the efficacious force of the pier to prevent its being overturned. And that the arch and pier may be just kept in equilibrium, we must make the force and resistance equal to each other, that is  $\frac{cdA}{b} = \frac{1}{2}exx$

$$= \frac{cdA}{b}. \text{ Hence then } x = \sqrt{\frac{2cdA}{eb}} =$$

$\sqrt{\frac{2AM \times AL \times A}{DQ \times EF}}$  will be the breadth or thickness of the pier required.

In the above investigation it is supposed that the whole of the pier was out of water: But if any part of it OL be supposed to be immersed in water, that part will lose so much of its weight as is equal to its bulk of water; and since the specific gravity of water is to that of common stone, as 1 is to  $2\frac{1}{2}$ , or as 2 to 5, it is evident that OL will lose 2 parts in 5 of its weight. Hence then, putting  $g = OG$ , since  $OG \times GL = gx$  is the area immersed, therefore  $\frac{2}{5}gx$  = the weight lost by the immersion; which being taken from  $ex$  the whole, we shall have  $ex - \frac{2}{5}gx$  as the weight remaining appended to E; then this being drawn into  $GE = \frac{1}{2}x$ , and the product equated to the efficacious force of the arch as before, we have  $\frac{1}{2}exx - \frac{2}{5}gxx = \frac{cdA}{b}$ ; and hence  $x = \sqrt{\frac{10cdA}{b \cdot 5e - 2g}}$

for the thickness of the pier when it is immersed in water to the height expressed by  $g$ .

—Or, because  $g$  will be nearly equal to  $d$ , the theorem for the thickness may be  $x = \sqrt{\frac{10cdA}{b \cdot 5e - 2d}} = \sqrt{\frac{10AM \times AL \times A}{DQ \times 5EF - 2AL}}$ .

*Corol-*

*Corollary 1.*

When DA is a quadrant, the arch is a complete femicircle; and then  $h$  is  $= r$ ,  $A = a + \frac{1}{14}r \times r$  as in Cor. 1 to prop. 8, and by the same Corollary  $c$  or  $r - QM$  is  $= r - \frac{3a+r}{a+\frac{1}{14}r} \times \frac{1}{2}r$   
 $= \frac{3a+\frac{1}{2}r}{a+\frac{1}{14}r} \times \frac{1}{2}r$ . Consequently  $cA$  is  $= \frac{3a+\frac{1}{2}r}{a+\frac{1}{14}r} \times \frac{1}{2}rr$ .

This value being substituted in the two preceding theorems, we have  $n = \sqrt{\frac{dr}{c} \times \frac{21a+2r}{21}}$   
 $= \sqrt{\frac{dr}{a+r+d} \times \frac{21a+2r}{21}} =$   
 $\sqrt{\frac{AL \times AQ}{IL} \times \frac{21DK+2AQ}{21}} =$  thickness of  
the pier when it is dry.—Or, if  $n$  express  
what part  $a$  is of  $r$ , or  $DK = \frac{1}{n}$ th of  $DQ$  or  $QA$ ,  
the same thickness will be  $r\sqrt{\frac{d}{21} \times \frac{2+2n}{r+r+d \cdot n}}$   
 $= AQ \times \sqrt{\frac{\frac{1}{14}AL \times \frac{21+2n}{AQ+AQ+AL \cdot n}}{}}$

And the thickness when  $AL$  is under water  
will be  $n = \sqrt{\frac{5dr}{5c-2d} \times \frac{21a+2r}{21}} =$

$$\sqrt{\frac{dr}{a+r+\frac{1}{7}d}} \times \frac{21a+2r}{21} =$$

$$\sqrt{\frac{AL \times AQ}{IL - \frac{1}{7}AL}} \times \frac{21DK + 2AQ}{21}. \text{--- Or, if } a$$

$$= \frac{r}{n} \text{ as before, the same thickness will be}$$

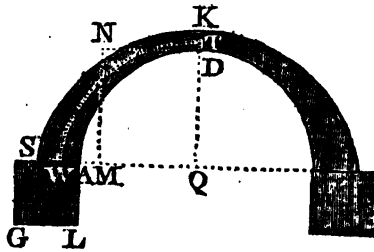
$$r \sqrt{\frac{d}{21} \times \frac{21+2n}{r+r+\frac{1}{7}d.n}} = AQ \times$$

$$\sqrt{\frac{\frac{1}{21}AL \times \frac{21+2n}{AQ+AQ+\frac{1}{7}AL.n}}{21}}.$$

*Corollary 2.*

WHEN HG is = BG in the last figure; then the arch and pier will be as in this annexed figure. And,  $e$  being then =  $d$ , the two general theorems will become

$$x = \sqrt{\frac{2cA}{b}} =$$



$$\sqrt{\frac{2A \times AM}{DQ}} \text{ for the thickness of the pier when}$$

dry, and  $x = \sqrt{\frac{10cA}{3b}} = \sqrt{\frac{10A \times AM}{3DQ}} =$  the thickness when under water.

So that, in this case, it makes no difference of whatever height LA the pier is to the springing  
I of

of the arch. For though the drift of the arch be increased with the length of the lever or height of the pier, the weight of the pier itself, which acts against it, is also increased in the same proportion.

*Scholium.*

IN the investigation of this proposition, the sections of the arch and pier are used for their solidities, as being evidently in the same proportion, or in that of their weights, since they are of the same length, viz. the breadth of the bridge.

By the above rules, together with those in the four preceding propositions, the necessary thickness of a pier may be found, so that it shall *just* balance the spread or shoot of the arch, independent of any other arch on the other side of the pier. But the weight of the pier ought a little to preponderate against or exceed in effect the shoot of the arch; and therefore the thickness ought to be taken a little more than what will be found by these rules; unless it be supposed that the pointed projections of the piers against the stream, beyond the common breadth of the bridge, will be a sufficient addition to the pier, to give it the necessary preponderancy.—But there is one very material thing, on account of which the thickness of the piers may be much diminished; viz. by the  
stones

stones of the wall above the voussoirs being bonded in with those of the pier and with one another, the pier will carry part of their weight; which will not only diminish the weight of the whole arch and wall, but will also both add the same to the weight of the pier, and lengthen the lever  $EG$ , by moving the center of gravity a little nearer to  $L$ ; but then also  $M$  will be a little nearer to  $Q$ , so that  $AM$  will be longer, and the effects of the change of the centers of gravity may be supposed nearly to balance each other.——In the foregoing propositions I have considered circular arches only, as it will make no difference of any consequence, to suppose the arches of any other curve of the same span and pitch. But this 10th prop. is general for all curves,

I shall now add a few examples of the calculation in numbers, to shew the manner, and in them also to point out the easiest methods of calculation.

#### EXAMPLE I.

SUPPOSING the arch in the figure to the proposition to be a semicircle whose height or pitch is 45 feet, and consequently its span 90 feet; also suppose the thickness  $DK$  at top to be 6 feet, and the height  $LA$  to the springing 18; and let it be required to find the thickness  $GL$ .

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of the pier necessary to resist the drift of the arch.

This will be immediately found by Cor. 1, in which  $AQ$  is = 45,  $AL$  = 18, and  $n = \frac{r}{a} = \frac{49}{6} = 7\frac{1}{2}$ .

Then the first expression  $AQ \times \sqrt{\frac{AL}{21} \times \frac{21 + 2n}{AQ + AQ + AL \cdot n}}$  will become  $\frac{540}{\sqrt{2415}} = 10.988$ , or 11 feet nearly for the thickness of the pier when dry.

And the latter expression  $AQ \times \sqrt{\frac{AL}{21} \times \frac{21 + 2n}{AQ + AQ + \frac{1}{2}AL \cdot n}}$  will give  $\frac{540}{\sqrt{2163}} = 11.61$  feet for the thickness when 18 feet are under water.

EXAMPLE 2,

In the same figure, suppose the span to be 100 feet, the height 40 feet; also the thickness at top 6 feet, and the height of the pier to the springer 18 feet as before.

Here the figure either is or may be considered as a scheme arch, or the segment of a circle, in which the versed sine  $QD$  is = 40, and the right sine  $QA$  = 50; also  $DK$  = 6,  $AL$  = 18,  $EF$  = 64.

Now

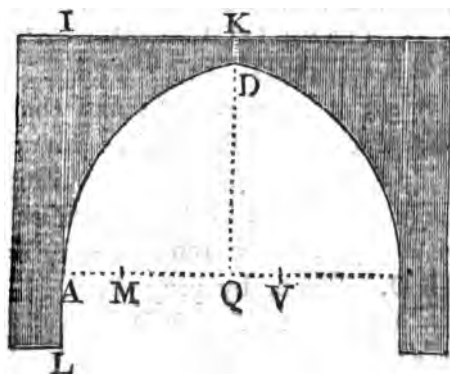
Now, by the nature of the circle, the radius  
 $VD = r$  is  $= \frac{QA^2 + QD^2}{2QD} = \frac{50^2 + 40^2}{80} = 51\frac{1}{4}$ ;  
 hence  $VQ = 51\frac{1}{4} - 40 = 11\frac{1}{4}$ ; and the area of  
 the semi-segment  $ADQ$  will be found to be  
 1490.9998, or 1491 nearly; which being taken  
 from the rectangle  $AIKQ = AQ \times QK =$   
 $50 \times 46 = 2300$ , there remains  $809 = A$  the  
 area  $AIKD$ . Then, by prop. 8,  $QM$  will be  
 $= \frac{VD^2 - VQ^2}{2A} \times VK = \frac{VD^2 - VQ^2}{3A} =$   
 $\frac{51.25^2 - 11.25^2}{2 \times 809} \times 57\frac{1}{2} = \frac{51.25^2 - 11.25^2}{3 \times 809} =$   
 $33.58$ ; and consequently  $MA = AQ - QM =$   
 $50 - 33.58 = 16.42$ .

Then, the first expression  $\sqrt{\frac{2AL \times AM \times A}{DQ \times EF}}$   
 will become  $\sqrt{\frac{36 \times 16.42 \times 809}{40 \times 64}} = 13.67$ , or  
 $13\frac{1}{2}$  feet nearly = the thickness of the pier when  
 dry.

And the latter expression  $\sqrt{\frac{10AL \times AM \times A}{DQ \times 5EF - 2AL}}$   
 will give  $\sqrt{\frac{180 \times 16.42 \times 809}{40 \times 320 - 36}} = 14.508$ , or  $14\frac{1}{2}$   
 feet nearly = the thickness when 18 feet are  
 under water.

Ex-

## EXAMPLE 3.



LET the arch be of the gothic kind, as in the annexed figure; in which DA is a circular arc whose center is V, its sine  $DQ = 50 =$  the height of the arch, its versed sine  $AQ = 40 =$  the semi-span, the thickness at top  $DK = 6$ , and the height AL of the pier to the spring = 18 as before.

Here the radius  $VA = 51\frac{1}{2}$  as in the last example, and the semi-segment  $ADQ = 1491$ , also the same as in the last example; then the rectangle IQ is  $= AQ \times QK = 40 \times 56 = 2240$ ; from which taking the semi-segment, there remains  $749 = A$  for the area AIKD. Then, by prop. 9, VM will be equal to  $\frac{3KD + DQ}{6A} \times DQ^2 = \frac{18 + 50}{6 \times 749} \times 50^2 = 37.83$ ; and hence  $MA = c = 51.25 - 37.83 = 13.42$ .

Then

Then the first expression  $\sqrt{\frac{2AL \times AM \times A}{DQ \times IL}}$   
 will become  $\sqrt{\frac{36 \times 13.42 \times 749}{50 \times 74}} = 9.889$ , or  
 nearly 10 feet for the thickness of the pier when  
 when it is all out of water.

And the latter one  $\sqrt{\frac{10AL \times MA \times A}{DQ \times 5IL - 2AL}}$  will  
 give  $\sqrt{\frac{180 \times 13.42 \times 809}{50 \times 370 - 36}} = 10.409$ , or  $10\frac{1}{2}$   
 nearly = the thickness when 18 feet are under  
 water.

#### EXAMPLE 4.

WHEN the arch stones only are laid, and the  
 pier built no higher than the spring, it will ap-  
 pear as in the figure to corollary 2. And then  
 if, in the first case, the arch be a complete  
 semicircle whose diameter is 90 feet, and the  
 thickness everywhere  $DK = AS = 6$  feet: It  
 is required to find the breadth of the piers.

The bounding arcs being quadrants, the area  
 $ADKS$  will be  $\frac{AD + KS}{2} \times DK = \frac{90 + 102}{2}$   
 $\times \frac{1}{4} \times 6 = 144 \times \frac{1}{4} = 452.4 = A$ . Now if  
 $TW$  be another concentric quadrant bisecting  
 the area  $ADKS$ , the center of gravity of  $TW$   
 may

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may be taken for that of the said area. And then, by the corollary to prop. 6, QM will be  $= \frac{7}{11}QT$ ; but since the quadrants QDA, QTW, QKS are in arithmetic progression, the squares of their semidiameters QD, QT, QK will be in the same progression, that is  $2QT^2 = QD^2 + QK^2$ , or  $QT = \sqrt{\frac{QD^2 + QK^2}{2}} = \sqrt{\frac{45^2 + 51^2}{2}} = 48.094$ ; hence then  $QM = \frac{7}{11}QT$  is  $= \frac{7}{11} \times 48.094 = 30.605$ , and consequently  $MA = 45 - 30.6 = 14.4$ .

Then the former of the two expressions in corollary 2 to this proposition, will give GL or  $\sqrt{\frac{2A \times AM}{DQ}} = \sqrt{\frac{904.8 \times 14.4}{45}} = 17.016$ , or 17 feet for the thickness of the pier when out of water.

And the latter one  $\sqrt{\frac{10A \times AM}{3DQ}}$  will become  $\sqrt{\frac{4524 \times 14.4}{135}} = 21.97$ , or nearly 22 feet for the thickness when the pier is immerfed in water.

*Scholium.*

OR, because QT is nearly an arithmetic mean between QD and QK, half the sum of QD and QK might have been used instead of it, without

without causing any sensible difference in the conclusion.

We might also exhibit general theorems for the thickness, in terms of the radius only.

For, taking  $QT$  or  $QW = \frac{QD + QK}{2}$ , by the corollary to prop. 6 we have  $QM = \frac{7}{11} QW$   
 $= \frac{QD + QK}{22} \times 7$ , and thence  $AM = c = AQ$   
 $- QM = QA - \frac{7QD + 7QK}{22} = \frac{8QD - 7DK}{22}$ .

Also  $A = \frac{AD + KS}{2} \times DK = \frac{QD + QK}{2} \times \frac{1}{7} DK$   
 $= \frac{2QD + DK}{14} \times 11 DK$ . Then these values

being substituted in the expression  $\sqrt{\frac{2A \times AM}{QD}}$

we shall have  $\sqrt{\frac{16QD^2 - 6QD \times DK - 7DK^2}{14QD}}$

$\times DK$  for the thickness of the pier when dry; and the same expression multiplied by  $\sqrt{\frac{1}{7}}$  will give the thickness when the pier is immersed in water. And, farther, if  $DK$  be assumed equal to any part of  $DQ$ , as  $DK = n \times DQ$ ; then the thickness in the former case will be  $QD \times$

$\sqrt{\frac{16 - 6n - 7nn}{14}} \times n$ , and in the latter  $QD \times$

$\sqrt{\frac{80 - 30n - 35nn}{42}} \times n$ .

K

Then

Then, by assuming several values of  $n$  from  $\frac{1}{15}$  to  $\frac{1}{7}$ , which are beyond the limits of it, the several breadths of the piers corresponding to the several values of the thickness of the arch, both when the pier is supposed to be out of water, and immerfed in it, will be found from these expreffions as in the following table; where the fractional part  $\frac{1}{7\frac{1}{2}}$  or  $\frac{2}{15}$  is also given, because it is the most common proportion.

*A Table of the Breadth or Thickness of a Pier answering to the several thicknesses of a semicircular arch, as in the foregoing example, QD being the radius or semi-span.*

For the pier dry		For the pier in water	
Thickness of the arch	Thickness of the pier	Thickness of the arch	Thickness of the pier
$\frac{1}{15}$ QD	·331 QD	$\frac{1}{15}$ QD	·427 QD
$\frac{1}{9}$ QD	·348 QD	$\frac{1}{9}$ QD	·449 QD
$\frac{1}{8}$ QD	·368 QD	$\frac{1}{8}$ QD	·475 QD
$\frac{1}{7\frac{1}{2}}$ QD	·379 QD	$\frac{1}{7\frac{1}{2}}$ QD	·488 QD
$\frac{1}{7}$ QD	·391 QD	$\frac{1}{7}$ QD	·505 QD
$\frac{1}{6}$ QD	·420 QD	$\frac{1}{6}$ QD	·542 QD
$\frac{1}{5}$ QD	·455 QD	$\frac{1}{5}$ QD	·588 QD

Ex-

## EXAMPLE 5.

BUT supposing the same figure in Cor. 2 to be a circular segment, whose chord or span is 100 feet, and height 40 feet, also the thickness of the arch 6 feet: To find the thickness of the piers.

Here the radius of the middle arc TW is  $\frac{QW^2 + QT^2}{2QT} = \frac{53^2 + 43^2}{86} = 54\frac{7}{11}$ ; hence TW is an arc of  $78^\circ 6'$ , and its length will be  $73.8293$ ; which being multiplied by  $DK = AS = 6$ , we have  $A = 442.9758$ . Then, by prop. 6, QM will be found  $= \frac{53^2 + 43^2}{2 \times 73.8293} = 31.545$ . And consequently  $AM = 50 - 31.545 = 18.455$ .

Hence, by Cor. 2, it will be  $\sqrt{\frac{2A \times AM}{DQ}} = \sqrt{\frac{885.9516 \times 18.455}{40}} = 20.218 =$  the thickness of the pier when dry,

And  $\sqrt{\frac{10A \times AM}{3DQ}} = \sqrt{\frac{4429.758 \times 18.455}{120}} = 26.101 =$  the thickness in water.



*Otherwise.*

BUT if the arch be supposed to increase in thickness from the top at D, where it is 6 feet, all the way to the spring, where it is AS = 12 feet suppose; the height and span being 40 and 100 as before.

Then QS = 62, QW = 56, and QT = 43. Hence the radius of the arc TW will be  $\frac{QW^2 + QT^2}{2QT} = \frac{56^2 + 43^2}{86} = 59.3452$ ; and therefore TW is an arc of  $70^\circ 40'$ , and its length =  $73.1945$ . Consequently the area ADKS or TW  $\times \frac{DK + AS}{2}$  will be  $73.1945 \times 9 = 658.75 = A$ . And, by prop. 6, QM will be  $\frac{56^2 + 43^2}{2 \times 73.1945} = 34.053$ ; and therefore AM =  $50 - 34.053 = 15.947$ .

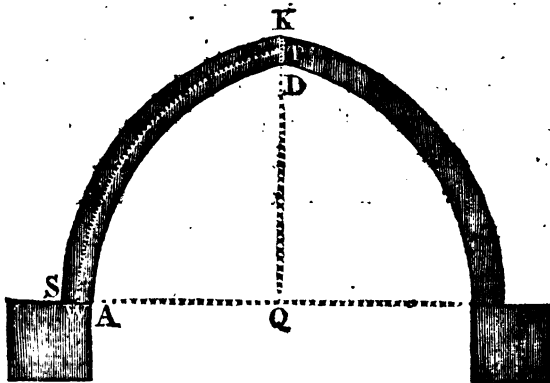
Hence, as above,  $\sqrt{\frac{13175 \times 15.947}{40}} = 22.918$  will be the thickness of the pier when dry.

And  $\sqrt{\frac{6587.5 \times 15.947}{120}} = 29.588 =$  the thickness in water.

Ex-

## EXAMPLE 6.

In a gothic arch whose thickness at top is 6, the span 80, and height 50 feet; to find the thickness of the piers.



By the last example, TW is =  $73.8293$ , its radius =  $54\frac{7}{11}$ , and the area ADKS =  $442.9758$ . Then, by prop. 7, we have  $QM = 43 - 54\frac{7}{11} + \frac{53 \times 54\frac{7}{11}}{73.8293} = 27.718$ ; and hence  $AM = 40 - 27.718 = 12.282$ .

Then, by Cor. 2, we shall have  $\sqrt{\frac{2 \times 442.9758 \times 12.282}{50}} = 14.752$  for the thickness of the pier when dry.

And  $\sqrt{\frac{442.9758 \times 12.282}{150}} = 19.045 =$  the thickness when in water.

Also

Also if the arch stones were supposed to lengthen all the way from the top towards the lower end, the calculation might be made as in the last example.

Having, in these 2d and 3d sections, gone through the calculations for the form of arches, and the thickness of piers; I shall now in the next section add some investigations of rules for determining the best form of the ends of the piers, with the force of the water upon them, &c.

## SECTION IV.

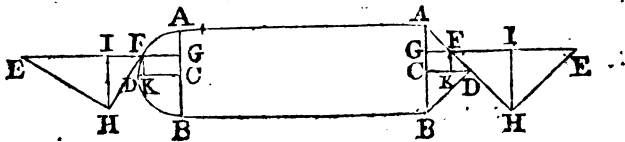
*The Force of the Water, &c.*

## PROPOSITION XI.

*To determine the form of the ends of a pier, so as to make the least resistance to, or be the least subject to the force of the stream of water.*

*Solution.*

LET the following figure represent a horizontal section of the pier, AB its breadth, CD the given length or projection of the end, and ADB the line required, whether right or curved; also let EF represent the force of a particle of water acting on AD at F in the direction parallel to the axe CD; produce EF to meet AB in G, and draw the tangent FH, also draw EH perpendicular to FH, HI perpendicular to EF, and FK perpendicular to DC.



Now

Now the absolute force EF of the particle of water may be resolved into the two forces EH, HF, and in those directions; of these the latter one, acting parallel to the curve, is of no effect; and the former EH is resolved into the two EI, IH; so that EI is the efficacious force of the particle to move the pier in the direction of its axe or length: That is, the absolute force is to the efficacious force, as EF is to EI.—Then, since EF is the diameter of a semicircle passing through H, by the nature of the circle we shall have  $EF : EI :: EF^2 : EH^2 ::$  (by similar triangles)  $HF^2 : HI^2$  and  $::$  the square of the fluxion of the curve or line : the square of the fluxion of the ordinate FK, because HF, HI are parallel to the line and ordinate.

Wherefore, putting the abscissa  $DK = x$ , the ordinate  $KF = y$ , and the line  $DF = z$ , we shall have as  $z^2 : y^2 :: 1$  (the force  $EF : \frac{y^2}{z^2}$ ) = the force of the particle at F to move the pier in the direction EFG. But the number of particles striking against the indefinitely small part of the line, is as  $y$ ; this drawn into the above found force of each, we have  $\frac{y^3}{z^2} = \frac{y^3}{x^2 + y^2}$  for the fluxion of the force, or the force acting against the part  $z'$  of the line.

But,

But, by the proposition, the whole force on DFA must be a minimum, or the fluent of

$\frac{y^3}{x^2 + y^2}$  must be a minimum when that of  $x$  becomes equal to the constant quantity DC;

in which case it will be found that  $\frac{xy^3}{x^2 + y^2}$  must be always equal to a constant quantity  $q$ ; and hence  $xy^3 = q \times \overline{x^2 + y^2}^2$ .

Now in this equation it is evident that  $x$  is to  $y$  in a constant ratio; but if two fluxions be always in a constant ratio, their fluents  $x$ ,  $y$ , are known to be also in a constant ratio, which is the property of a right line.

Wherefore DFA is a right line, and the end ADB of the pier must be a right-lined triangle, that the force of the water upon it may be the least possible.

## PROPOSITION XII.

To determine the resistance of the end of a pier against the stream of water.

*Solution.*

Using here the figure and notation of the last proposition, by the same it is found that the fluxion of the force of the stream against the face

DF is  $\frac{y^3}{x^2 + y^2}$ ; and since the fluxion of the

force against the base is  $y$ , it follows that the force of the stream against the base AB is to the force against the face ADB, as  $(y)$  the fluent

of  $y$  is to the fluent of  $\frac{y^3}{x^2 + y^2}$ . That is, the

the absolute force of the stream is to the efficacious force against the face of the pier, as its

breadth is to double the fluent of  $\frac{y^2}{x^2 + y^2}$  when  $y$  is equal to half the breadth.

*Corollary 1.*

If the face ADB be rectilineal.

Putting  $DC = a$ ,  $CA = b$ , and  $AD = (\sqrt{aa + bb} =) c$ ; as  $a : b :: x : y$  by similar triangles;

Sect. IV. *Force of the Water, &c.*

triangles; hence  $x = \frac{ay}{b}$ , and  $\dot{x} = \frac{a\dot{y}}{b}$ ; this being written for it in the general expression above, we have  $\frac{y^2}{\frac{a^2 y^2}{b^2} + y^2} = \frac{bb\dot{y}}{aa + bb} = \frac{bb\dot{y}}{cc}$  for the

fluxion of the force on AD; the fluent of which, or  $\frac{bb\dot{y}}{cc}$ , is the force itself. And consequently the force on the flat base AB is to that on the triangular end, as  $y$  to  $\frac{bb\dot{y}}{cc}$ , or as  $cc$  to  $bb$ , that is, as  $AD^2$  to  $AC^2$ .

And if AC be equal to CD, or ADB a right angle, which is generally the case, then  $AD^2 = 2 AC^2$ , and the force on the base to that on the face, as 2 to 1.

Moreover, as the force on ADB, when ADB is a right angle, is only half of the absolute force, so it is evident that the force will be more than one-half when ADB is greater than a right angle, and less when it is less; and also that the longer AD is, the less the force is, it being always inversely as the square of AD.



## Corollary 2.

If ADB be a semicircle.

The radius  $AC = CD = a$ ; then  $2ax - xx$   
 $= yy$ , or  $x = a - \sqrt{aa - yy}$ , and  $\dot{x} = \frac{yy}{\sqrt{aa - yy}}$ ;  
 hence  $\frac{\dot{y}^2}{x^2 + y^2}$  becomes  $\frac{\dot{y}^2}{\frac{y^2 \dot{y}^2}{aa - yy} + \dot{y}^2} = \frac{aa - yy}{aa}$

$\times \dot{y}$ , the fluent of which is  $\frac{aa - \frac{1}{3}yy}{aa} \times y$ ; and  
 therefore the force on the base is to the force  
 on the circular end, as  $y$  is to  $\frac{aa - \frac{1}{3}yy}{aa} \times y$ , or  
 as  $aa$  to  $aa - \frac{1}{3}yy$ , or as  $3aa$  to  $3aa - yy$ .

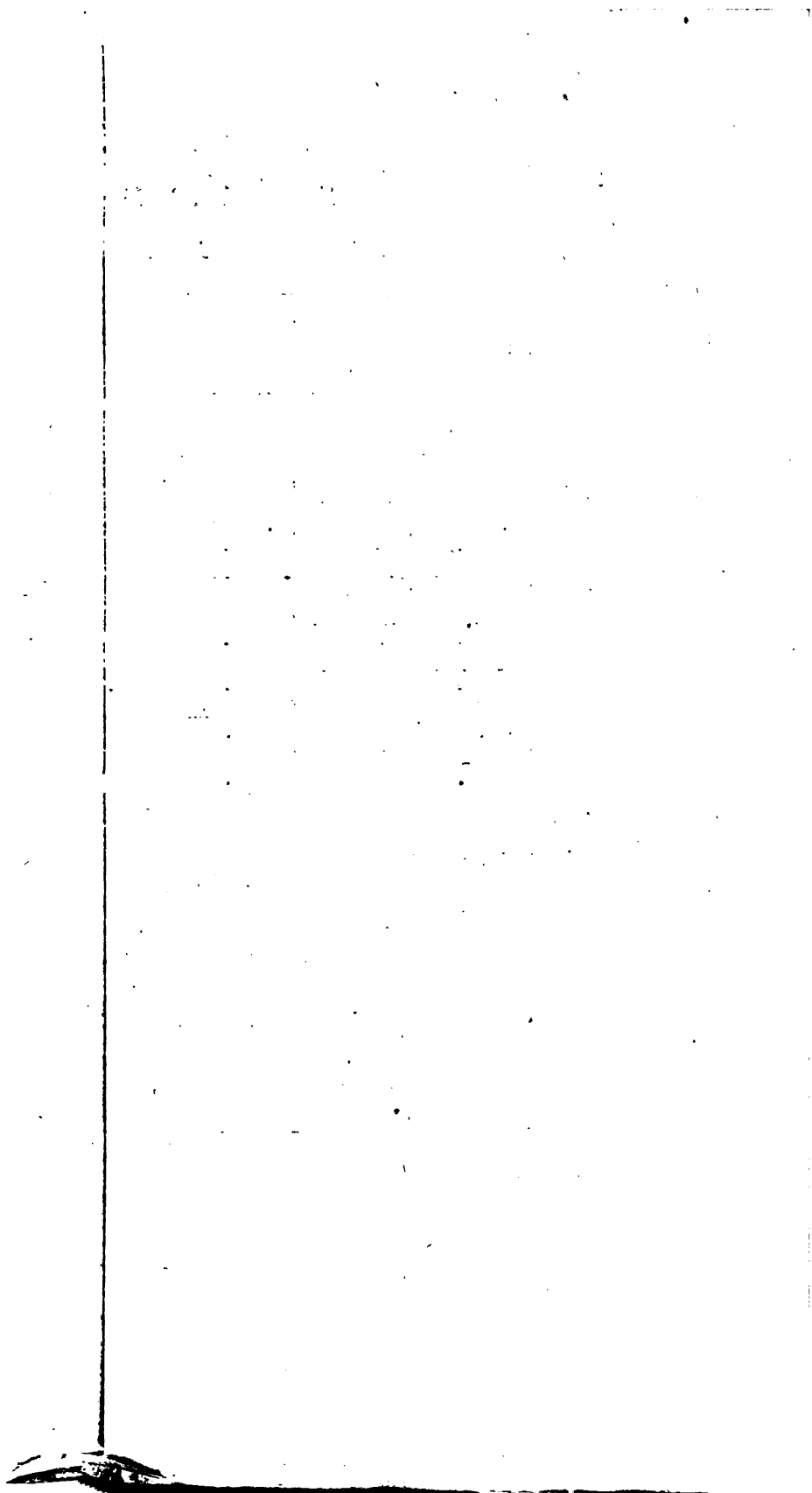
And when  $y = a = AC$ , the proportion be-  
 comes that of 3 to 2.

So that only one-third of the absolute force  
 is taken off by making the end a semicircle.

## Corollary 3.

WHEN the face ADB is a parabola.

Then, the notation being as before, viz.  $DC$   
 $= a$ , and  $AC = b$ , we have  $a : bb :: x : yy$ ;  
 hence  $x = \frac{ayy}{bb}$ , and  $\dot{x} = \frac{2ay\dot{y}}{bb}$ ; which being  
 written



, in Proportion to the  
l, in its Passage.

		Stages of Accumulation in Floods	Construction of an ancient bridge of 3 or more arches.
7-8ths			Resistance 5-18ths
Parts.			Rise of Water
ts.	F. I. Pts.	Uniform Tenors.	F. I. Pts.
41	1 4 .728		0 0 .320
64	5 6 .9		0 1 .28
68	12 6 .53	Ordinary Floods.	0 2 .881
56	22 3 .6		0 5 .119
84	34 10 .31	Extraordinary Floods.	0 8 .003
76	50 2 .112		0 11 .525
10	Piers 140	Torrents above generally Inunda- tions.	Piers 50
60	Arches 160		River 180
ped to rise above its natural level, passage; therefore these numbers true height of the flood.—The construction, and in all states of a dvantage of bridges of a sufficient ridge is nearly in the 6th predica- se the Thames, with a velocity of city of 2.5 f. per second, to only			In this most common mode seldom suffici- ent in a flood, the water soon encroaches on the arches, and changes the predicament.

This table to face page 77.

written in the general expression, the fluent of it becomes the circular arc whose radius is  $\frac{bb}{2a}$  and tangent  $y$ ; and so the absolute force is to the force on the parabolic end, as  $y$  to the arc whose tangent is  $y$  and radius  $\frac{bb}{2a}$ ; that is, as the tangent of an arc is to the arc itself, the radius being to the tangent as 2 to  $\frac{bb}{ay}$ . And when  $y = b$ , the ratio of the tangent to radius, is that of 2 to  $\frac{b}{a}$ ; or that of 2 to 1 when  $DC = CA$ .

In which case the whole force is to the force on the parabolic end, as the tangent is to the arc of which the tangent is double the radius; that is, as the tangent of  $63^{\circ} 26' 4''$  to the arc of the same, or as 2 to 1.10714; which is a less force than on the circle, but greater than on the triangle.

And so on for other curves; in which it will be found that the nearer they approach to right lines, the less the force will be, and that it is least of all in the triangle, in which it is one-half of the whole absolute force when right-angled.

The annexed folding-out sheet shews at one view the rise of the water under the arches arising from its obstruction by the piers, according to several rates of velocity, &c.

SEC-

## SECTION V.

*Of the Terms or Names of the various parts peculiar to a Bridge, and the Machines, &c. used about it; disposed in alphabetical order,*

**ABUTMENT, or BUTMENT,** which see in its place below.

**ARCH,** an opening of a bridge, through or under which the water, &c. passes, and which is supported by piers or by butments.

Arches are denominated circular, elliptical, cycloidal, catenarian, &c. according to the figure of the curve of them. There are also other denominations of circular arches according to the different parts of a circle: So, a semicircular arch is half the circle; a scheme or skreen arch is a segment less than the semicircle; and arches of the third and fourth point, or gothic arches, consist of two circular arcs, excentric and meeting in an angle at top, each being 1-3d or 1-4th, &c. of the whole circle.

The chief properties of the most considerable arches, with regard to the extrados they require, &c. may be learned from the second section. It there appears that none, but the arch  
of

of equilibration in the example to prop. 5, can admit of a horizontal line at top; that this arch is not only of a graceful but of a convenient form, as it may be made higher or lower at pleasure with the same opening; that it, but no other, with a horizontal top, can be equally strong in all its parts, and therefore ought to be used in all works of much consequence. All the other arches require tops that are curved either upward or downward, some more and some less: Of these the elliptical arch seems to be the fittest to be substituted instead of the equilibrial one with any tolerable degree of propriety; it is in general also the best form for most bridges, as it can be made of any height to the same span, or of any span to the same height, while at the same time its hanches are sufficiently elevated above the water, even when it is pretty flat at top; which is a property of which the other curves are not possessed in the same degree; and this property is the more valuable, because it is remarked that after an arch is built and the centering struck, it settles more about the hanches than the other parts, by which other curves are reduced near to a straight line at the hanches. Elliptical arches also look bolder, are really stronger, and require less materials and labour than the others. Of the other curves, the cycloidal arch is next in quality to the elliptical one, for all the above properties. And, lastly, the circle. As to the others, the  
para-

parabola, hyperbola, and catenary, they may not at all be admitted in bridges of several arches ; but may in some cases be used for a bridge of one single arch which is to rise very high, because then not much loaded at the hanches. We may hence also perceive the falsity of those arguments which assert, that because the catenarian curve supports itself equally in all its parts, it will therefore best support any additional weight laid upon it : for the additional building made to raise the bridge to a horizontal line, or nearly such, by pressing more in one part than another, must force those parts down, and the whole must fall. Whereas other curves will not support themselves at all without some additional parts built above them, to balance them, or to reduce their parts to an equilibrium.

**ARCHIVOLT**, the curve or line formed by the upper sides of the voussoirs or arch stones. It is parallel to the intrados or underside of the arch when the voussoirs are all of the same length ; otherwise not.

By the archivolt is also sometimes understood the whole set of voussoirs.

**BANQUET**, the raised foot path at the sides of the bridge next the parapet. This ought to be allowed in all bridges of any considerable  
size.

size : it should be raised about a foot above the middle or horse passage, made 3, 4, 5, 6, 7, &c. feet broad according to the size of the bridge, and paved with large stones whose length is equal to the breadth of the walk.

**BATTARDEAU**, or *Coffer-dam*, a case of piling, &c. without a bottom, fixed in the bed of the river, water-tight or nearly so, by which to lay the bottom dry for a space large enough to build the pier on. When it is fixed, its sides reaching above the level of the water, the water is pumped out of it, or drawn off by engines, &c. till the space be dry ; and it is kept so by the same means, if there are leaks which cannot be stopped, till the pier is built up in it ; and then the materials of it are drawn up again.

Battardeaux are made in various manners, either by a single inclosure, or by a double one, with clay or chalk rammed in between the two, to prevent the water from coming through the sides. And these inclosures are also made either with piles only, driven close by one another, and sometimes notched or dove-tailed into each other ; or with piles grooved in the sides, driven in at a distance from one another, and boards let down between them in the grooves.



The method of building in battardeaux cannot well be used where the river is either deep or rapid. It also requires a very good natural bottom of solid earth or clay; for, although the sides be made water-tight, if the bottom or bed of the river be of a loose consistence, the water will ooze up through it in too great abundance to be evacuated by the engines.

It is almost needless to remark that the sides must be made very strong, and well propt or braced in the inside, to prevent the ambient water from pressing the sides in, and forcing its way into the battardeau.

BRIDGE, a work of carpentry or masonry, built over a river, canal, &c. for the convenience of crossing the same.

A stone bridge is an edifice forming a way over a river, &c. supported by one arch or by several arches, and these again supported by proper piers or buttments.

A stately bridge over a large river is one of the most noble and striking pieces of art. To behold huge and bold arches, composed of an immense quantity of small materials, as stones, bricks, &c. so disposed and united together that they

they seem to form but one solid compact body, affording a safe passage for men and carriages over large waters, which with their navigation pass free and easy under them at the same time, is a sight truly surprizing and affecting indeed.

To the absolutely necessary parts of a bridge already mentioned, viz. the arches, piers, and abutments, may be added the paving at top, the parapet wall, either with or without a balustrade, &c. also the banquet or raised foot way on each side, leaving a sufficient breadth in the middle for horses and carriages. The breadth of a bridge for a great city should be such as to allow an easy passage for three carriages and two horsemen a-breast in the middle way, and for three foot passengers in the same manner on each banquet. And for other less bridges a less breadth.

As a bridge is made for a way or passage over a river, &c. so it ought to be made of such a height as will be quite convenient for that passage; but yet so as to be consistent with the interest and concerns of the river itself, easily admitting through its arches the craft that navigate upon it, and all the water even at high tides and floods. The neglect of this precept has been the ruin of many bridges, and particularly that at Newcastle, over the river Tyne, on the 17th of november 1771. So that in de-

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termining its height, the conveniencies both of the passage over it and under it should be considered; and the height made to answer the best for them both, observing to make the *convenient* give place to the *necessary* when their interests are opposite.

Bridges are generally placed in a direction perpendicular to the stream in a direct line, to give free passage to the water, &c. But some think they should be made not in a straight line, but convex towards the stream, the better to resist floods, &c. And some such bridges have been made.

Again, a bridge should not be made in too narrow a part of a navigable river, or one subject to tides or floods: because the breadth being still more contracted by the piers, will increase the depth, velocity, and fall of the water under the arches, and endanger the whole bridge and navigation.

The number of arches of a bridge are generally made odd; either that the middle of the stream or chief current may flow freely without the interruption of a pier; or that the two halves of the bridge, by gradually rising from the ends to the middle, may there meet in the highest and largest arch; or else, for the sake of grace, that by being open in the middle, the eye in viewing

viewing it may look directly through there, as one always expects to do in looking at it, and without which opening one generally feels a disappointment in viewing it.

If the bridge be equally high throughout, the arches, being all of a height, are made all of a size; which causes a great saving of centering. If the bridge be higher in the middle than at the ends, let the arches decrease from the middle towards each end, but so as that each half have the arches exactly alike, and that they decrease in span, proportionally to their height, so as to be always the same kind of figure, and similar parts of that figure: thus, if one be a semicircle, let the rest be semicircles also, but proportionally less; if one be a segment of a circle, let the rest be similar segments of other circles; and so for other figures. The arches being equal at equal distances on both sides of the middle, is not only for the strength and beauty of the bridge, but that the centering of the one half may serve for the other also. But if the bridge be higher at the ends than in the middle, the arches ought to increase in span and pitch from the middle towards the ends.

When the middle and ends are of different heights, their difference however ought not to be great in proportion to the length, that the ascent may be easy; and then also it is more beautiful

beautiful to make the top one continued curve than two inclined straight lines from the ends towards the middle.

Bridges should rather be of few and large arches than of many and small ones, if the height and situation will possibly allow of it; for this will leave more free passage for the water and navigation, and be a great saving in materials and labour, as there will be fewer piers and centers, and the arches themselves will require less materials.

For the fabric of a bridge, and the proper estimation of the expence, &c. there are generally necessary three plans, three sections, and an elevation. The three plans are so many horizontal sections, viz. the first a plan of the foundation under the piers, with the particular circumstances attending it, whether of gratings, planks, piles, &c. the second is the plan of the piers and arches, &c. and the third is the plan of the superstructure, with the paved road and banquet. The three sections are vertical ones; the first of them a longitudinal section from end to end and through the middle of the breadth; the second a transverse one, or across it, and through the summit of an arch; and the third also across, and taken upon a pier. The elevation is an orthographic projection of one side or face of the bridge, or its appearance as viewed at  
at

at an infinite distance, and shews the exterior aspect of the materials, and the manner in which they are worked and decorated.

Other observations are to be seen in the first section.

**BUTMENTS**, or *abutments*, the extremities of a bridge, by which it joins to or abuts upon the land or sides of the river, &c. These must be made very secure, quite immovable, and more than barely sufficient to resist the drift of its adjacent arch. So that if there are not rocks or very solid banks to raise them against, they must be well reinforced with proper walls or returns, &c. The thickness of them that will be barely sufficient to resist the shoot of the arch, may be calculated as that of a pier by prop. 10.

When the foundation of a butment is raised against a sloping bank of rock, gravel, or good solid earth, it will produce a saving of materials and labour, to carry the work on by returns at different heights, like steps of stairs.

**CAISSON**, a kind of *chest*, or flat-bottomed boat, in which a pier is built, then sunk to the bed of the river, and the sides loosened and taken off from the bottom, by a contrivance for that purpose; the bottom of it being left  
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under the pier as a foundation. It is evident therefore that the bottoms of caissons must be made very strong and fit for the foundations of the piers. The caisson is kept a float till the pier be built to about the height of low-water mark ; and for that purpose its sides must either be made of more than that height at first, or else gradually raised to it as it sinks by the weight of the work, so as always to keep its top above water. And therefore the sides must be made very strong, and kept asunder by cross timbers within, lest the great pressure of the ambient water crush the sides in, and so not only endanger the work, but also drown the men who work within it. The caisson is made of the shape of the pier, but some feet wider on every side to make room for the men to work : the whole of the sides are of two pieces, both joined to the bottom quite around, and to each other at the salient angle, so as to be disengaged from the bottom and from each other when the pier is raised to the desired height, and sunk. It is also convenient to have a little sluice made in the bottom, occasionally to open and shut, to sink the caisson and pier sometimes by, before it be finished, to try if it bottom level and rightly ; for by opening the sluice, the water will rush in and fill it to the height of the exterior water, and the weight of the work already built will sink it ; then by shutting the sluice again, and pumping out the water,

water, it will be made to float again, and the rest of the work may be completed: but it must not be sunk but when the sides are high enough to reach above the surface of the water, otherwise it cannot be raised and laid dry again. Mr. Labelye tells us that the caissons in which he built some of the piers of Westminster bridge, contained above 150 load of fir timber of 40 cubic feet each, and was of more tonnage or capacity than a 40 gun ship of war.

CENTERS, are the timber frames erected in the spaces of the arches to turn them on, by building on them the vouffoirs of the arch. As the center serves as a foundation for the arch to be built on, when the arch is completed, that foundation is struck from under it, to make way for the water and navigation, and then the arch will stand of itself from its curved figure. A center must therefore be constructed of the exact figure of the intended arch, convex as the arch is concave, to receive it on as a mould. If the form be circular, the curve is struck from a central point by a radius: if it be elliptical, it ought to be struck with a doubled cord, passing over two pins or nails fixed in the focusses, as the mathematicians describe their ellipses; and not by striking different pieces or arcs of circles from several centers; for these will form no ellipse at all, but an irregular misshapen curve made up of broken pieces of dif-

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ferent circular arcs: but if the arch be of any other form, the several abscissas and ordinates ought to be calculated, then their corresponding lengths, transferred to the centering, will give so many points of the curve, and exactly by which points bending a bow of pliable matter, the curve may be drawn by it.

The centers are constructed of beams, &c. of timber firmly pinned and bound together, into one entire compact frame, covered smooth at top with planks or boards to place the voussloirs on, the whole supported by offsets in the sides of the piers, and by piles driven into the bed of the river, and capable of being raised and depressed by wedges, contrived for that purpose, and for taking them down when the arch is completed. They ought also to be constructed of a strength more than sufficient to bear the weight of the arch.

In taking the center down; first let it down a little, all in a piece, by easing some of the wedges; there let it rest a few hours or days to try if the arch make any efforts to fall, or any joints open, or stones crush or crack, &c. that the damage may be repaired before the center is entirely removed, which is not to be done till the arch ceases to make any visible efforts.

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In some bridges the centering makes a very considerable part of the expence, and therefore all means of saving in this article ought to be closely attended to; such as making few arches, and as nearly alike or similar as possible, that the centering of one arch may serve for others, and at least that the same center may be used for both of each pair of equal arches on both sides of the middle.

CHEST, the same as *Caïsson*.

COFFERDAM, the same as *Battardeau*.

DRIFT; *Shoot*, or *Thrust* of an arch, is the push or force which it exerts in the direction of the length of the bridge. This force arises from the perpendicular gravitation of the stones of the arch, which, being kept from descending by the form of the arch and the resistance of the pier, exert their force in a lateral or horizontal direction. This force is computed in prop. 10, where the thickness of the pier is determined that is necessary to resist it; and is greater the lower the arch is, *cæteris paribus*.

ELEVATION, the orthographic projection of the front of a bridge on the vertical plane parallel to its length. This is necessary to shew the form and dimensions of the arches and other

parts as to height and breadth, and therefore has a plain scale annexed to it to measure the parts by. It also shews the manner of working up and decorating the fronts of the bridge.

**EXTRADOS**, the exterior curvature or line of an arch. In the propositions of the second section it is the outer or upper line of the wall above the arch; but it often means only the upper or exterior curve of the voussiors.

**FOUNDATIONS**, the bottoms of the piers, &c. or the bases on which they are built. These bottoms are always to be made with projections, greater or less according to the spaces on which they are built. And according to the nature of the ground, depth and velocity of water, &c. the foundations are laid and the piers built after different manners, either in caissons, in battardeaux, on stilts with sterlings, &c. for the particular methods of doing which, see each under its respective term.

The most obvious and simple method of laying the foundations and raising the piers up to water-mark, is to turn the river out of its course above the place of the bridge, into a new channel cut for it near the place where it makes an elbow or turn; then the piers are built on dry ground, and the water turned into its old course again, the new one being securely banked up.

up. This is certainly the best method, when the new channel can be easily and conveniently made; but which however is seldom or never the case.

Another method is to lay only the space of each pier dry till it be built, by furrounding it with piles and planks driven down into the bed of the river, so close together as to exclude the water from coming in; then the water is pumped out of the inclosed space, the pier built in it, and lastly the piles and planks drawn up. This is coffer-dam work, but evidently cannot be practised if the bottom be of a loose consistence admitting the water to ooze and spring up through it.

When neither the whole nor part of the river can be easily laid dry as above, other methods are to be used; such as to build either in caissons or on stilts, both which methods are described under their proper words; or yet by another method, which hath, though seldom, been sometimes used, without laying the bottom dry, and which is thus: the pier is built upon strong rafts or gratings of timber well bound together, and buoyed up on the surface of the water by strong cables, fixed to other flotes or machines, till the pier is built; the whole is then gently let down to the bottom, which must be made level for the purpose. But of these

these methods, that of building in caissons is the best.

But before the pier can be built in any manner, the ground at the bottom must be well secured, and made quite good and safe if it be not so naturally. The space must be bored into to try the consistence of the ground; and if a good bottom of stone, or firm gravel, clay, &c. be met with within a moderate depth below the bed of the river, the loose sand, &c. must be removed and digged out to it, and the foundation laid on the firm bottom on a strong grating or base of timber made much broader every way than the pier, that there may be the greater base to press on, to prevent its being sunk. But if a solid bottom cannot be found at a convenient depth to dig to, the space must then be driven full of strong piles, whose tops must be sawed off level some feet below the bed of the water, the sand having been previously digged out for that purpose; and then the foundation on a grating of timber laid on their tops as before. Or, when the bottom is not good, if it be made level, and a strong grating of timber, two, three, or four times as large as the base of the pier be made, it will form a good base to build on, its great size preventing it from sinking. In driving the piles, begin at the middle, and proceed outwards all the way to the borders or margin: the reason of which is, that  
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if the outer ones were driven first, the earth of the inner space would be thereby so jammed together, as not to allow the inner piles to be driven. And besides the piles immediately under the piers, it is also very prudent to drive in a single, double, or triple row of them around and close to the frame of the foundation, cutting them off a little above it, to secure it from slipping aside out of its place, and to bind the ground under the pier the firmer. For, as the safety of the whole bridge depends on the foundation, too much care cannot be used to have the bottom made quite secure.

**JETTEE**, the border made around the flits under a pier, being the same with *Sterling*.

**IMPOST**, is the part of the pier on which the feet of the arches stand, or from which they spring.

**KEYSTONE**, the middle vouffoir, or the arch stone in the top or immediately over the center of the arch. The length of the keystone, or thickness of the archivolt at top, is allowed to be about 1-15th or 1-16th of the span, by the best architects.

**ORTHOGRAPHY**, the elevation of a bridge, or front view as seen at an infinite distance.

**PARAPET**,

**PARAPET**, the breast wall, made on the top of a bridge to prevent passengers from falling over. In good bridges, to build the parapet but a little part of its height close or solid, and upon that a balustrade to above a man's height, has an elegant effect.

**PIERS**, the walls built for the support of the arches, and from which they spring as their bases.

They ought to be built of large blocks of stone, solid throughout, and cramped together with iron, which will make the whole as one solid stone. Their faces or ends, from the base up to high-water mark, ought to project sharp out with a salient angle, to divide the stream. Or, perhaps, the bottom of the pier should be built flat or square up to about half the height of low-water mark, to allow a lodgment against it for the sand and mud, to cover the foundation; lest, by being left bare, the water should in time undermine and so ruin or injure it. The best form of the projection for dividing the stream, is the triangle; and the longer it is, or the more acute the salient angle, the better it will divide it, and the less will the force of the water be against the pier; but it may be sufficient to make that angle a right one, as it will make the work stronger, and in that case the  
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perpendicular projection will be equal to half the breadth or thickness of the pier. In rivers on which large heavy craft navigate and pass the arches, it may perhaps be better to make the ends semicircular ; for although it does not divide the water so well as the triangle, it will both better turn off and bear the shock of the craft.

The thickness of the piers ought to be such as will make them of weight or strength sufficient to support their interjacent arch independent of any other arches. And then if the middle of the pier be run up to its full height, the centering may be struck to be used in another arch before the hanches are filled up. The whole theory of the piers may be seen in the third section.

They ought to be made with a broad bottom on the foundation, and gradually diminished in thickness by offsets up to low-water mark.

The methods of laying their foundations, and building them up to the surface of the water, are given under the word FOUNDATION.

PILES, are timbers driven into the bed of the river for various purposes, and are either round, square, or flat like planks. They may be of any wood which will not rot under water, but oak and fir are mostly used, especially the latter, on account of its length, fireightness, and

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cheapness. They are shod with a pointed iron at the bottom, the better to penetrate into the ground ; and are bound with a strong iron band or ring at top, to prevent them from being split by the violent strokes of the ram by which they are driven down.

Piles are either used to build the foundations on, or are driven about the pier as a border of defence, or to support the centers on ; and in this case, when the centering is removed, they must either be drawn up or sawed off very low under water ; but it is perhaps better to saw them off and leave them sticking in the bottom, lest the drawing of them out should loosen the ground about the foundation of the pier. Those to build on, are either such as are cut off by the bottom of the water, or rather a few feet within the bed of the river ; or else such as are cut off at low-water mark, and then they are called *stilts*. Those to form borders of defence, are rows driven in close by the frame of a foundation, to keep it firm ; or else they are to form a cafe or jettée about *stilts*, to keep within it the stones that are thrown in to fill it up ; in this case, the piles are grooved, driven at a little distance from each other, and plank piles let into the grooves between them, and driven down also, till the whole space is surrounded. Besides using this for *stilts*, it is also sometimes necessary to surround a stone pier with

with a sterling or jettée, and fill it up with stones to secure an injured pier from being still more damaged, and the whole bridge ruined. The piles to support the centers may also serve as a border of piling to secure the foundation, cutting them off low enough after the center is removed.

**PILE DRIVER**, an engine for driving down the piles. It consists of a large ram of iron sliding perpendicularly down between two guide posts; which being lift up to the top of them, and there let fall from a great height, comes down upon the top of the pile with a violent blow. It is worked either with men or horses, and either with or without wheel work. That which was used at the building of Westminster bridge, is perhaps the best ever invented.

**PITCH**, of an arch, the perpendicular height from the spring or impost to the keystone.

**PLAN**, of any part, as of the foundations, or piers, or superstructure, is the orthographic projection of it on a plane parallel to the horizon.

**PUSH**, of an arch, the same as drift, shoot, &c.

**SALIENT ANGLE**, of a pier, the projection of the end against the stream, to divide it. The right-lined angle best divides the stream, and the more acute the better for that purpose; but the right angle is generally used as making the best masonry. A semicircular end, though it does not divide the stream so well, is sometimes better in large navigable rivers, as it carries the craft the better off, or bears their shocks the better.

**SHOOT**, of an arch, the same as drift.

**SPRINGERS**, are the first or lowest stones of an arch, being those at its feet bearing immediately on the impost.

**STERLINGS**, or *Jettées*, a kind of case made about a pier of stilts, &c. to secure it, and is particularly described under the next word *Stilts*.

**STILTS**, a set of piles driven into the space intended for the pier, whose tops being sawed level off about low-water mark, the pier is then raised on them. This method was formerly used when the bottom of the river could not be laid dry; and these stilts were surrounded, at a few feet distance, by a row of piles and planks, &c. close to them like a coffer-dam, and called a sterling or jettée; after which loose stones,

stones, &c. are thrown or poured down into the space till it be filled up to the top, by that means forming a kind of pier of rubble or loose work, and which is kept together by the fides or sterlings: this is then paved level at the top, and the arches turned upon it. This method was formerly much used, most of the large old bridges in England being erected that way, such as London bridge, Newcastle bridge, Rochester bridge, &c. But the inconveniencies attending it are so great, that it is now quite exploded and disused: for, because of the loose composition of the piers, they must be made very large or broad, or else the arch would push them over and rush down as soon as the center was drawn; which great breadth of piers and sterlings so much contracts the passage of the water, as not only very much incommodes the navigation through the arch, from the fall and quick motion of the water, but from the same cause also the bridge itself is in much danger, especially in time of floods, when the water is too much for the passage. Add to this that besides the danger there is of the pier bursting out the sterlings, they are also subject to much decay and damage by the velocity of the water and the craft passing through the arches.

THRUST, the same as drift, &c.

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**VOUSOIRS**, the stones which immediately form the arch, their under sides constituting the intrados. The middle one, or keystone, ought to be about 1-15th or 1-16th of the span, as has been observed; and the rest should increase in size all the way down to the impost; the more they increase the better, as they will the better bear the great weight which rests upon them without being crushed, and also will bind the firmer together. Their joints should also be cut perpendicular to the curve of the intrados,

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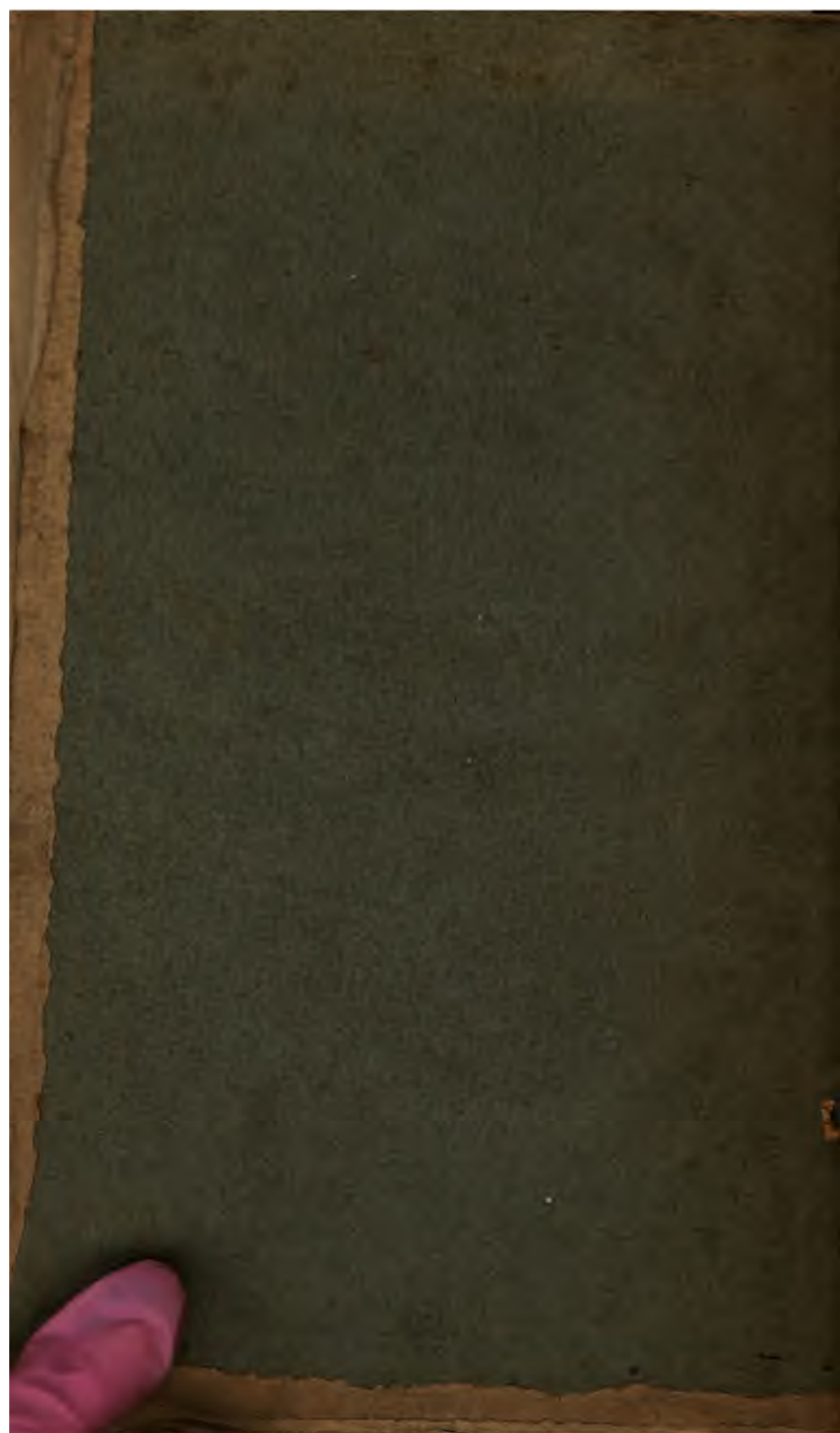
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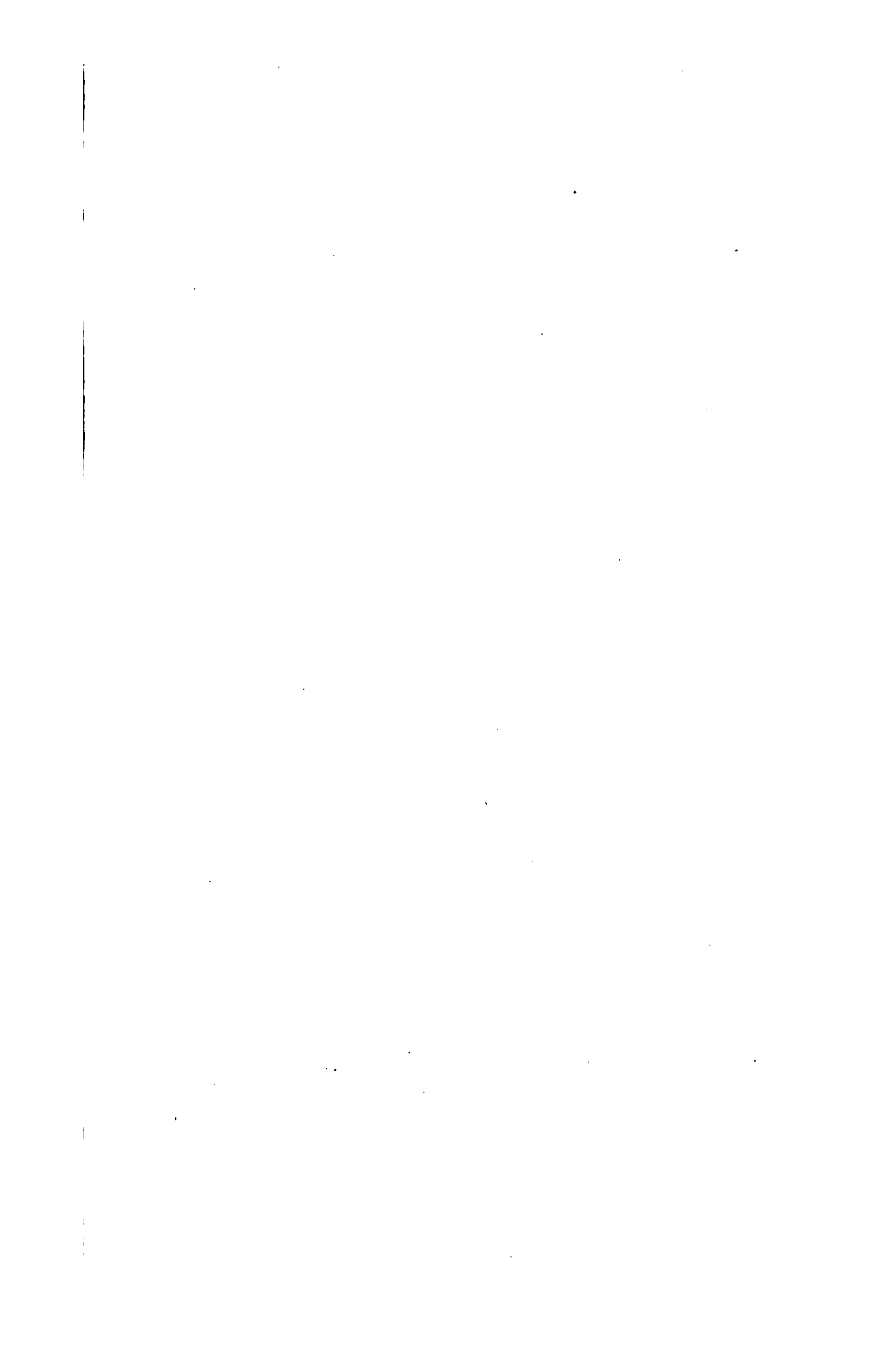
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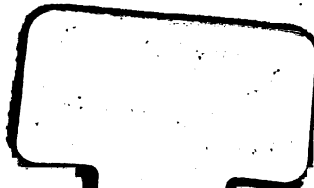


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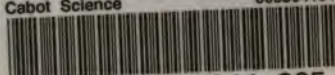
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